# 2 Mathematical approaches to rhythm used in the twentieth-century

THE OPENING CHAPTER included an exploration of the ground-breaking rhythmic techniques used by Stravinsky and Bartók. This chapter examines the work of further twentieth-century composers who took innovative approaches to rhythm rooted in mathematics, in particular Henry Cowell, Conlon Nancarrow, John Cage, Olivier Messiaen and Steve Reich. It opens by exploring the music and writings of the American composer, music publisher and teacher Henry Cowell whose work foreshadowed some of the main developments in twentieth-century music, not least in rhythm. Cowell outlined the majority of his ideas about music in his seminal work *New Musical Resources (1938),* and many of them were later taken up by composers in America and Europe. Nancarrow's rhythms were so complex that, inspired by the writing of Cowell, he composed most of his music for the automated player piano. Cage, a one-time pupil of Cowell, experimented with rhythm through the use of what he referred to as square root form. The next section explores Messiaen's approach to rhythm based on his principle of added values and this is followed by Reich's rhythmic transformation through phase shifting. The chapter closes with an example of hyper-complexity in the music of Brian Ferneyhough moving on to the potential of computer-generated rhythms.

## Henry Cowell's experiments in rhythm

In 1919 Cowell composed *Fabric*, a short piano piece based on the interactions of various rhythmic units based on different numbers. For example, in bar one the lowest part in the left hand is written in straightforward semi-

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quavers (sixteenth notes) whilst above it the middle part uses a rhythmic unit of five and the top part is divided into two units of three. In the preface to this piece Cowell (1897-1965) includes an 'Explanation of New Rhythms and Notes' in which he advocates a new system of rhythmic notation that he had expounded on in his book *New Musical Resources*[20]. Rather than always subdividing note lengths by two (whole note, half note, quarter notes and so on) he proposes that irregular time values 'be called by their correct names according to the part of a whole note they occupy'. Thus triplets would be referred to as third notes, quintuplets would be fifth notes and so on up to fifteenth notes. In order to be able to distinguish these new types of notes, the note heads would be different shapes, the third note series, for example, would have triangular note heads and the fifth notes series would have square note heads.

Cowell completed two quartets around the same time, *Quartet Romantic* (1917) and *Quartet Euphometric* (1919). Both were rhythmic experiments, the first in its extreme rhythmic complexity and the second in its use of 'implied or actual polytempo or polymeter'. In his prefatory notes Cowell described them as 'totally impractical works at that time' which, because of their rhythmic complications, could only be performed by 'electronic means'. He looked forward to a time when a keyboard instrument could be devised which would be able to control more complex rhythms.[21] Ten years later he met the electrical engineer Leon Theremin, now best known as the inventor of his namesake instrument the *Theremin* which he had recently created. Together Cowell and Theremin invented the Rhythmicon an instrument that could play triplets against quintuplets, or any other combination of up to sixteen notes in a group.[22] When a single note was held down a steady rhythm was produced whereas holding down multiple notes produced polyrhythms. The instrument was operated by a keyboard producing a percussive almost drum-like sound. The rhythms were related to the corresponding frequency of the vibrations so, for example, quintuplets would be sounded on the fifth harmonic, and nonuplets on the ninth harmonic. All of this meant that Cowell had to create a new system of notation for its use. By the end of 1931

20 Henry Cowell. *New Musical Resources. (New York: Alfred a Knopf, 1930): 58.*

21 David Nicholls and Joel Sachs. 'Henry Cowell' in *Grove Music.*

22 For further details of this instrument please see Leland Smith. 'Henry Cowell's Rhythmicana' in *Anuario Interamericano de Investigacion Musical,* Vol. 9 (1973): 134-147.

he had written *Rhythmicana*, a four-movement piece for rhythmicon and orchestra. For various reasons, *Rhythmicana* did not receive its premiere until 40 years later when it was performed by the Stanford Symphony Orchestra in 1971 using a computer.[23]

Rather confusingly, Cowell produced a four-movement piano work with the same name in 1938. *Rhythmicana* for piano provides a further illustration of Cowell's rhythmic experimentation. The first two movements have the time signature of 1/1 and the third has a time signature of 5/4 in the left hand and 3/4 in the right hand meaning that the barlines in the left and right hand staves are in different places. There are examples of conflicting polyrhythms throughout.

## Conlon Nancarrow and the player piano

Much of the early piano music of the American-Mexican composer Conlon Nancarrow (1912-1997) was so rhythmically complex that it was almost impossible to perform. When Nancarrow came across a copy of Cowell's *New Musical Resources* (1930), he read that some rhythms 'could not be played by any living performer; but these highly engrossing rhythmic complexes could easily be cut on a player-piano roll.'[24] In 1947 Nancarrow travelled to New York where he bought one of the instruments and had a roll-punching machine built. From then on Nancarrow composed music with highly intricate rhythmic designs for the auto-playing piano. His *Studies for Player Piano* comprise 50 pieces covering a range of rhythmic techniques, mostly derived from four basic rhythmic ideas: ostinato, isorhythm, tempo canon and acceleration.

Nancarrow was a jazz trumpeter and much of his music has a jazzy feel. His first pieces, the group known as Study No. 3, were extremely fast jazz pieces, influenced by Art Tatum and Earl Hines. In Nos. 1, 2, 3, 5 and 9, ostinatos are set against each other at different speeds. Nos. 6, 7, 10, 11 and 20 use the medieval technique of isorhythm with multiple repetition of the same

23 One of the original machines can be found in the Smithsonian Institute, Washington, DC but it is no longer operational. An operational model is housed at the Theremin Centre for Electroacoustic Music in Moscow.

24 Cowell, *New Musical Resources*, 65.

rhythm against different pitch patterns, sometimes played at lightning speed.

Tempo canons are used in Nos. 13–19. The word canon refers to a form or a compositional technique (see pages 141-52). Canons are based on the principle of imitation, in which an initial melody is imitated at a specified time interval by one or more parts and in a tempo canon the same idea is used at different speeds. Nancarrow takes a melody or block of texture and superimposes it upon itself at varying tempo ratios, so for example a ratio of 4: 5 could indicate that one melody, going at a speed of 100 beats per minute (bpm), is played with the same melody superimposed at a speed of 125 bpm. At some point the melodies converge at the same point in their material, so for example, in No. 14 the convergence point falls at the mid-point of the piece. In Nancarrow's later canonic studies the ratios became more complex: in No. 33 the ratio is $\sqrt{2}$: 2 and in No. 40 it is $e$: $\pi$ ($e$ being the base of natural logarithms), so in other words roughly 2.718: 3.142.

Studies 8, 21, 22, 23, and 27 to 30 are based on different rates of acceleration and deceleration; in No. 27 for example, the voices of the canon accelerate and decelerate at rates of 5%, 6%, 8% and 11%. Notes are shortened to achieve different rates of acceleration; an 11% acceleration would mean that each note is 11% shorter than its predecessor. Nancarrow combined these various rhythmic ideas in his later studies and also utilised other features that the player piano was capable of such as the extremely fast glissandi and arpeggios in No. 27 and, at the climax of No. 25, 1028 notes whizz past in 12 seconds.[25]

## John Cage and square root form

For a brief period in 1933, Cage studied with Henry Cowell whom he described as "the open sesame for new music in America" a reference to both Cowell's pioneering ideas about music and his role as a music publisher promoting new composers.[26] It has been argued that from Cowell Cage

25 Kyle Gann. 'Conlon Nancarrow' in *Grove Music*.

26 John Cage. *Silence. Lectures and writings by John Cage.* (Hanover, New England: Wesleyan University Press, 1973): 67.

'inherited a spirit of musical adventurousness'.[27]

In the 1930s Cage began to experiment with the form of his pieces, believing that the structure of a piece should be defined by rhythm rather than harmony and melody. He was later to write:

> Sound has four characteristics: pitch, timbre, loudness and duration. The opposite and necessary coexistent of sound is silence. Of the four characteristics of sound, only duration involves both sound and silence. Therefore a structure based on durations (rhythmic: phrase, time, lengths) is correct (corresponds with the nature of the material), whereas harmonic structure is incorrect (derived from pitch, which has no being in silence).[28]

This led to his concept of the micro-macrocosmic rhythmic structure in which each part of a composition (microstructure) was divided into the same proportions as those of the piece as a whole (macrostructure). Cage sometimes referred to this as 'square root form'. In his piece *First Construction (in Metal)* (1939) the structure is 4: 3: 2: 3:4, a total of 16 units where each unit is represented by a bar of 4/4. The 16-bar structure is heard 16 times.

Cage's one-time teacher Cowell experimented with the piano strings being plucked, scratched and manipulated in other ways in, for example, his solo piano piece *Banshee*. Cage started similar experiments, discovering that by placing a variety of objects (for example metal screws, bolts, washers and nuts, plastic and cloth) between the strings the piano could be made to sound more like a percussion ensemble.

In 1945 he composed *Three Dances for Two Prepared Pianos*, a virtuoso piece for two amplified prepared pianos.[29] The piano preparation in *Three Dances* involves 36 notes on each piano, and uses a variety of materials including screws, pennies, rubber, plastic, weather stripping, and various bolts and nuts. The piece was premiered in New York in 1945 where it was

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27 Ed. David Nicholls. *The Cambridge Companion to John Cage*. (Cambridge: Cambridge University Press, 2002): 16.

28 Cited in *The Cambridge Companion to John Cage*, 246.

29 A piano which has been prepared by altering the pitches, timbres and dynamic responses of individual notes by means of bolts, screws, mutes and/or other objects inserted at particular points between or next to the strings.

used as the music for a dance piece, 'Dromenon', by Cage's lifelong partner the choreographer Merce Cunningham. Dance No. 1 is 30 x 30 bars long with a rhythmic structure where the 30 bars are divided as follows - 2 : 5 : 2 : 2 : 6 : 2 : 2 : 7 : 2. These divisions are marked by boxed numbers. The piece uses many repetitions, and the sections and phrases are differentiated by changes in timbre and shifts from one rhythm to another. In addition, the proportion for each dance is defined for a particular tempo with a change in tempo causing change in the proportion. Dance No. 1 has a tempo of 88 minim beats per minute whereas Dance No. 2 has a tempo of 114 minim beats per minute; the ratios are adjusted proportionately so that there are 39 x 39 bars with a rhythmic structure of 3 : 6 : 3 : 3 : 7 : 3 : 3 : 8 : 3.

Rhythmically *Three Dances* is very intricate. In the polymetric opening bars, for example, where the two pianos use a different metre; Piano 2 has four crotchet beats in the left hand and off beat quavers in the right hand, whereas Piano 1 is made up of groups of three quavers which go across the bar line and do not fall on any of the main beats. The percussive sounds and fast interlocking patterns are reminiscent of the Indonesian gamelan (see pages 45-7).

Cage wrote several pieces for the prepared piano over the 1940s and 1950s including the large-scale concert work *Sonatas and Interludes*. Here, for the first time he used rhythmic structures using fractions. *Sonata 1* is made up of 7 x 7 bars where the units are arranged according to the proportions 1 ¼ : ¾ : 1 ¼ : ¾ : 1 ½ : 1 ½.[30]

## Olivier Messiaen and his use of rhythm

'Let us not forget that the first, essential element in music is rhythm, and that rhythm is first and foremost the change of number and duration', said Olivier Messiaen in his lecture at the 1958 'Conference de Bruxelles' echoing Cage's words on the importance of rhythm.[31] In Messiaen's book *The Technique of My Musical Language* (1944 he outlines his compositional methods.[32]

30 Nicholls, *The Cambridge Companion to John Cage, 82.*

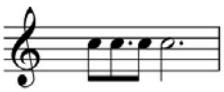
31 These words are taken from a lecture given by Messiaen at the 'Conferences de Bruxelles' in 1958.

32 Olivier Messiaen. *Technique of My Musical Language* (Paris: Alphonse Leduc, 1944).

He devotes a large part of his writing to rhythm with chapters on Hindu rhythm, rhythms with added values, augmented and diminished rhythms, nonretrogradable rhythms, polyrhythm and rhythmic pedals. His interest in rhythm stemmed from his student days in the 1920s, a significant moment being his discovery of a table of 120 *decitalas* – Hindu rhythms which had been transcribed by the thirteenth-century Hindu theorist Carngadeva.[33] [34]Messiaen was fascinated by the characteristics of these *talas* (repeated cyclical rhythmic patterns) and the ways in which they were used. As Julian L Hook explains in his article about rhythm in the music of Messiaen

> Indian rhythms do not arise by elaboration of an underlying pulse in the manner of Western rhythms, but by accumulations of small rhythmic values. The concept of a beat is replaced, in effect, by a smallest note value …of which all other rhythmic values are multiples, often in irregular groupings.[35]

In the opening section on rhythm in *The Technique of My Musical Language* Messiaen takes one of these rhythms - *ragavardhana* - and uses it to demonstrate some of the qualities he had found in Hindu music. Here is the *tala* in its original form.

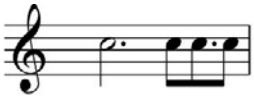


33 Messiaen found these Hindu rhythms in Lavignac's *Encyclopedie de la musique*.

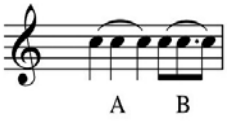
34 A *tala* is a repeated cyclical rhythmic pattern.

35 Julian L. Hook. 'Rhythm in the Music of Messiaen: An Algebraic Study and an Application in the "Turangalîla Symphony"'. *Music Theory Spectrum,* Vol. 20, No. 1 (Spring, 1998): 97-120.

Here is *ragavardhana* in reverse.



Messiaen points out that when *ragavardhana* is reversed, it contains the equivalent of (A) three crotchets (quarter notes) and (B) three quavers (eighth notes) and that B is an inexact diminution of A with the addition of a half-value (dot) on the second note (see below).

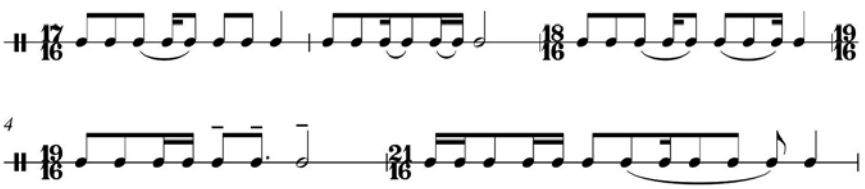



From this he derives the principle of 'added values' whereby a value can be added to a note by either lengthening it, or adding a short note-value or rest. Thus his approach to rhythm arises from an extension of durations (additive) rather than from a division of time (divisive).[36] As he observes, 'We can conclude [that] it is possible to add to any rhythm whatsoever a small brief value which transforms its metric balance'.[37] He went on to adapt these characteristics for use in his own music, altering, elaborating and superimposing them to create his own rhythmic language. Sometimes he used *ragavardhana* itself as in the organ piece *Les corps glorieux* and *Visions de l' Amen* for two pianos.

36 Robert Sherlaw Johnson. *Messiaen* (London, Omnibus Press, 2009).

37 Messiaen, *Technique of My Musical Language*, 14.

During World War II Messiaen was called up for military service and in 1940 he was captured and interred in a prisoner-of-war camp at Görlitz in Silesia. It was there that he completed the *Quatuor pour la fin du temps* (Quartet for the end of time) and performed it: Messiaen played the piano part, alongside three of his fellow inmates, a violinist, cellist and a clarinettist. In the Preface to this piece Messiaen describes the sixth movement, 'Danse de la fureur, pour les sept trompettes', as the most rhythmically characteristic movement. This is evident in its use of augmentations and diminutions, added values and the use of a series of non-retrogradable rhythms. Here is the rhythm of the first four bars which are played in unison, or at the octave, by the quartet (see below).



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Rather than constantly changing time signatures Messiaen abandons them altogether, only adding barlines to break up the blank succession of durations. The time signatures have been added in the quotation above for clarity, but they are not in the original score; the movement deliberately avoids a sense of metre. The note values do not follow conventional groupings, another device used to avoid the placement of accents and any consequent metrical implications. Each of the bars is of a different length but is based on the same rhythmic motifs, various combinations of quavers and semi-quavers. Bar 2 could be regarded as an inexact diminution of bar 1 in terms of the number of notes, whereas bar 5 is an inexact augmentation of bar 1.

Messiaen was fascinated by what he referred to as the 'charm of impossibilities', musical devices which included non-retrogradable rhythms and modes of limited transposition (see pages 113-16). In his own words, the charm of impossibilities 'resides particularly in certain mathematical

impossibilities [in] the modal and rhythmic domains' describing the rhythms as those 'which cannot be used in retrograde, because in such a case one finds the same order of values again'.[38] The first seven notes of bar 1 of 'Danse de la fureur' form a non-retrogradable rhythm – they are the same backwards as forwards.

In his *Technique of My Musical Language* Messiaen states that '[Canons] may exist without the presence of any melodic canon' and goes on to discuss examples of rhythmic canons by augmentation and diminution, along with nonretrogradable rhythms.[39] In 'La Bouscarle' (Cetti's Warbler) from Book 5 of his piano work *Catalogue d'Oiseaux* he uses a rhythmic canon in bars 20-24 where the left hand enters a semi-quaver beat after the right hand. The left hand part is an augmentation of the right hand part by a semi-quaver. Messiaen also used rhythmic canons in 'Regard du Fils sur le Fils' the fifth movement of *Vingt Regards sur l'enfant-Jésus* and the last movement of his song cycle *Harawi*.

In 1931 Messiaen heard a Balinese gamelan at the Exposition Coloniale in Paris. Gamelan is a term applied to both Indonesian orchestras, largely comprising gongs and metallophones, and to the music composed for them based on the overall effect of interlocking rhythms (see pages 45-7). The sound of the gamelan later prompted him to compose pieces for tuned percussion ensembles and was an important element of his ten-movement work *Turangalîla-symphonie* (1948). *Turangalîla* uses a huge battery of percussion instruments along with piano and *ondes martenot* (an early electronic instrument). Again there were influences from Indian classical music: the word *Turangalîla* itself is derived from Sanskrit vocabulary and can be loosely translated as 'love song', and three of the movements were originally subtitled *tala*. Most striking is his treatment of the percussion instruments which are used in the manner of the gamelan, not only in the gamelan-like sonorities produced by the tuned percussion (piano, glockenspiel, celeste and vibraphone) but also in the intricate textures; superimposed layers involving many simultaneous rhythmic processes.

38 Messiaen, *Technique of My Musical Language*, 16.

39 Messiaen, *Technique of My Musical Language*, 63.

## György Ligeti, *Poème Symphonique* and 100 metronomes

In 1962, the Hungarian composer György Ligeti (1923-2006) completed his *Poème Symphonique* where 100 mechanical metronomes are set to different speeds – between the metronome markings MM 144 and MM 50 - and then wound up for 'four half turns'.[40] They are then set in motion as 'simultaneously as possible' and the 'performers' leave the stage, leaving the metronomes to wind down. The LCM of the fastest speed (144 beats per minute) and the slowest speed (50 beats per minute) is 3600. This means that the first time the two metronomes will be heard together is on beat 3600. This takes place 25 minutes into the piece. Ligeti writes in the set of instructions which form the score:

> The overall design of the piece consists of a single long phrase which could be characterized as a rhythmic diminuendo: at the outset the number of metronomes ticking is so large that, heard together, the sound appears to be continuous. As the first metronomes come to rest, the static uniform sound thins and it gradually becomes possible for complex rhythms to be carved out of the now crumbling sound block. These rhythmic structures become increasingly clear as more and more instruments wind down: as the complexity is reduced, the rhythmic differentiation increases… towards the end of the work with the rhythmic differentiation further reduced, the rhythmic pattern becomes more regular… until only one metronome is left ticking.[41]

Or as Alex Ross puts it 'as the faster metronomes wind down and stop, spiderwebs of rhythm emerge from the cloud of ticks'.[42]

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40 A metronome is a device used to indicate the tempo of a composition by sounding regular beats at different speeds. Metronome markings are given in the score and indicate the number of beats per minute (bpm) for a specified note value e.g. a crotchet at MM = 120 would signify 120 crotchet beats per minute. The original metronomes were pyramid shaped, operated by clockwork and based on the principle of a double pendulum. Nowadays they are commonly digital electronic devices. The metronome was invented in 1814 by Dietrich Nikolaus Winkler (c.1780-1838) in Amsterdam. Two years later Johann Nepomuk Maelzel, an inventor of mechanical gadgets, copied the idea and patented it under his name. Beethoven was a fan of metronomes and was the first significant composer to adopt metronome markings.

41 Ligeti's programme notes from the score of *Poème Symphonique*.

42 Ross, Alex. *The Rest is Noise*. (London: Fourth Estate, 2008): 119.

## Steve Reich and phase shifting

The musical style known as Minimalism originated in America in the 1960s with composers including Terry Riley (see page 185), Steve Reich and Philip Glass. Minimalist music features constantly repeated patterns that are subjected to gradual processes of melodic and rhythmic transformation as the piece unfolds. It often uses interlocking repeated phrases and rhythms, addition and subtraction (where notes are added to or taken away from a repeated phrase), a layered texture and diatonic harmony. Central to Minimalism is a musical device known as phase shifting, constantly repeated patterns that are subjected to gradual changes, one part repeats constantly and another gradually shifts out of phase with it. The effect of phase shifting can be almost hypnotic. Phase shifting can be seen in terms of a mathematical translation or shift (see pages 30-4). By taking a periodic graph function (a function which repeats regularly in cycles such as a sine wave) and shifting it either horizontally or vertically a new function is created where although the general shape does not change, the point at which the wave starts does change. This type of transformation is known as phase shift.[43] The scientist Philip Ball, author of *The Music Instinct*, describes how

> … the voices interlock into distinct rhythmic patterns at different stages, which then dissolve and crystallize into new ones ..we form an interpretation of what the rhythmic pattern is, only to have to keep revising it as a new structure emerges.[44]

Ball goes on to compare this process with the Moiré patterns generated by overlapping grids. Moiré patterns are produced by two identical grids one of which is rotated relative to the other. As the rotation proceeds we see a series of regular geometric patterns formed from the grid lines that move in and out of phase with one another.[45]

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43 Gareth Roberts. *From Music to Mathematics {Exploring the connections}*. (Baltimore: John Hopkins University Press, 2016): 269.

44 Philip Ball. *The Music Instinct: How Music Works and Why We Can't Do Without It*. (London: Vintage, 2011): 223.

45 Ibid.

Steve Reich was born in New York in 1936 and started work as a composer in the 1960s. As a young man he became interested in the music of Africa, initially through A.M. Jones's work *Studies in African Music*. He was struck by the methods of music organisation which were radically different from those in the West.[46] He noted in particular the use of short repeating patterns played simultaneously but without their downbeats coinciding and the 'many rhythmic ambiguities inherent in subdivisions of 12.'[47]

Reich made his first forays into phase shifting in the 1960s in *It's Gonna Rain* (1965) and *Come Out* (1966) which both use recorded speech. He takes a single phrase or sentence and repeats it against versions of itself, looped and played at different speeds. On the *Come Out* tape, a Harlem teenager recounts being brutalized by officers for a crime he did not commit. Because he was not visibly bleeding he was turned away when he went to hospital for treatment. In his words "I had to, like, open the bruise up, and let some of the bruise blood come out to show them." Reich re-recorded the four-second fragment "come out to show them" on two channels. They begin in unison then slip out of sync to produce a phase shifting effect, the discrepancy widening to become at first a reverberation before splitting into four, then eight, until the words become unintelligible. Reich wrote "by not altering its pitch or timbre, one keeps the original emotional power that speech has while intensifying its melody and meaning through repetition and rhythm."[48] In 1967 he composed *Piano Phase* (1967) for two pianos (or piano and tape) which used a similar method in which the second player gradually makes slight rhythmic shifts with occasional re-alignments of the twelve notes against each other.

In 1971 Reich made a field trip to Ghana to study African drumming. On his return home he described the impression that the trip had had on him, saying that it 'turned out to be confirmation of gradually shifting phase relations between identical repeating patterns that I had used in *Piano Phase*.' Inspired by what he had learnt in Africa he went on to write *Drumming* (1971) which he described as the 'final expansion and refinement of the phasing process'. *Drumming* has one basic rhythmic pattern throughout

46 A M Jones. *Studies in African Music*. (London: Oxford University Press, 1959).

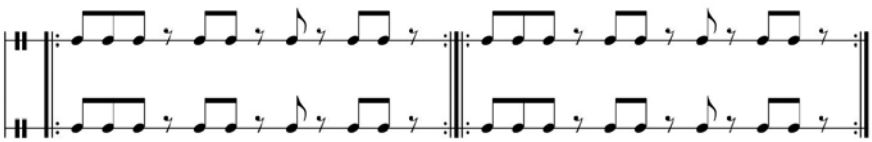
47 https://stevereich.com/composition/drumming/

48 Steve Reich wrote this in the liner notes to his album *Early Works, 1987*, which includes *Come Out and It's Gonna Rain*.

with the pattern undergoing changes of phase position, pitch, and timbre. In Reich's words the piece

begins with two drummers building up the basic rhythmic pattern of the entire piece from a single drum beat, played in a cycle of 12 beats with rests on all the other beats. Gradually additional drumbeats are substituted for the rests, one at a time, until the pattern is completed. The reduction process is simply the reverse where rests are gradually substituted for the beats… until only a single beat remains.[49]

*Clapping Music* (1972) is one of Reich's most performed and discussed pieces. It too uses phase shifting: two performers clap the same short 12-quaver phrase in unison as the piece opens. While one performer claps the same rhythm throughout the piece, the other claps the same sequence, but after 12 repetitions per bar, Player 2 shifts the emphasis by starting one quaver later, gradually moving through the whole pattern, quaver by quaver, and shifting further out of phase until eventually the parts come together again. The effect is one of huge rhythmic diversity. Here are the opening bars.



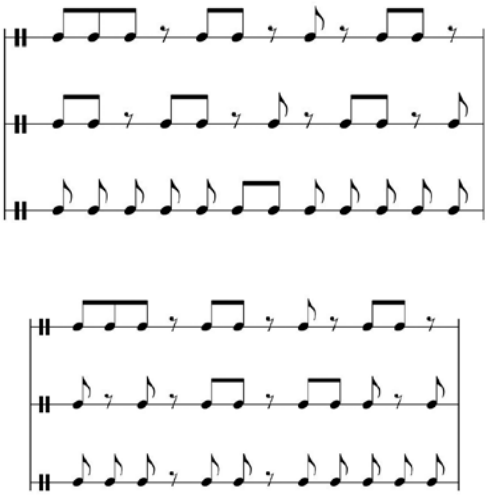


As he writes in the directions for performers, Reich has deliberately not included a time signature to avoid metrical accents. Rather he writes that the second player should always place their downbeat on the first beat of the new pattern and that no other accents should be made. This means that it is difficult to discern a clear downbeat at the beginning of each bar.

The original rhythm pattern has been carefully chosen in order that it has certain characteristics. Each bar of *Clapping Music* contains 12 'beats'.

49 https://stevereich.com/composition/drumming/.

Player 1 repeats the same bar throughout but with Player 2 the rhythm undergoes a cyclic shift or phase shift. In the second bar, the entire pattern is moved by a quaver to the left and the first note is placed at the end. The bar-long pattern could be denoted by the numbers 3, 2, 1, 2 (3 quavers, a rest, 2 quavers, a rest, 1 quaver, a rest and 3 quavers, a rest). This means that the fundamental pattern is asymmetric so that each new bar created by the shifting transformations is different to each other bar including the original. This creates 12 distinct patterns for Player 2 each having a different relationship with the original. So, for example, when Player 2 moves to the first shift, Players 1 and 2 combined results in eight quaver notes without any rests.

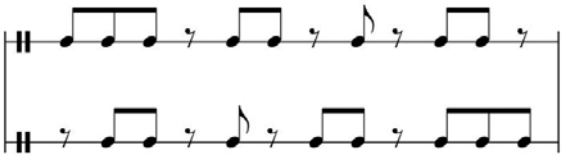


In contrast, a different musical character is produced when the rests coincide, for example bar 6.

Here the combined rhythm of the two players is quite different with what could be described perhaps as a dance-like feel.

Although the original rhythm pattern is asymmetric, the 12 new patterns produced by phase shifting produce symmetries with the original. So, for example, bar 4 which produces the pattern 2, 1, 2, 3, is a retrograde of the original (3, 2, 1, 2). In fact there are six further retrograde pairings to be

found in the piece.[50]



## Brian Ferneyhough and rhythmic complexity

The British composer Brian Ferneyhough, whose music belongs to the genre sometimes referred to as 'New Complexity', consistently uses highly complex rhythmic textures. The term New Complexity is used to categorize the aesthetic and formal characteristics of Ferneyhough's music as well as that of Michael Finnissy and a number of younger British composers. In the words of Christopher Fox 'they sought to achieve … a complex, multi-layered interplay of evolutionary processes occurring simultaneously within every dimension of the musical material…Microtonal pitch differentiations, ametric rhythmic divisions and the minutiae of timbral and dynamic inflection were all painstakingly notated…'.[51] Ferneyhough has referred to his use of complex rhythmic textures as shapes within shapes or hyper-rhythms. Here he explains his approach:

> Rhythm, metre and density are all different aspects of the disposition of events in time. If you have a set of measures, let's say 5/8, 3/8, 4/8, I may in the 5/8 measure take subdivisions of 5, 3 and 4. That set of proportions then anticipates the sequence of measure lengths (5, 3, 4) on the larger scale. Inside that 5, 3 and 4 it's possible to embed 5, 3 and 4, so that I have three levels of 5, 3 and 4. Even the additions of rests, for instance: …I say, 'Take five notes, leave out the next three notes, play four notes, leave out the next five notes', I am creating a hyper-

50 The American mathematician Joel K Haack set out a series of mathematical problems to be used in the classroom in his article 'Clapping Music-A Combinatorial Problem' in *The College Mathematics Journal* of San Jose State University, Vol 22, No 3, May 1991.

51 Christopher Fox in 'New Complexity' in *Grove Music*.

rhythm.[52]

The rhythmic complexity of Ferneyhough's music makes it very difficult to perform, hence the number of its performances is limited. As the composer Curtis Roads writes in *Rhythmic Processes in Electronic Music* (2014), 'In the late 20th century, rhythmic notation evolved over time into hyper-complexity, testing the limits of readability and playability. However, at the same time, the 'technology of electronic music made the design of complex rhythms ever more accessible.' He argues that 'technology has changed the paradigm of rhythmic theory and organization' facilitating 'the exploration of polyrhythmic grids and fields' which have led into 'uncharted rhythmic territories'.[53] In doing so it has spawned a multitude of new styles from contemporary art music (such as electroacoustic) to pop music (such as electronic dance music EDM with artists such as Andy C and Roni Size). In breakbeat-driven genres such as drum 'n' bass, for example, the rhythm is manipulated in various ways including asymmetrical rhythm divisions made possible by computer-driven technology (see pages 16-17). This does not mean that electronic music has replaced instrumental or vocal music, rather it has opened up a new set of possibilities.

This chapter has explored rhythmic innovations created by some of the leading twentieth-century composers of contemporary art music in the West. In several cases their influences have been drawn from other parts of the world, in particular India and Africa. The chapter which follows looks at North Indian classical music and West African drumming in more detail, along with music from other regions of the world, each with a singular approach to rhythm rooted in mathematics.

52 The words of Brian Ferneyhough cited in Michael Oliver. *Settling the Score: A Journey Through the Music of the Twentieth Century*. (London: Faber and Faber, 1999): 119).

53 Curtis Roads. *Rhythmic Processes in Electronic Music* Proceedings ICMC|SMC|2014 14-20 September 2014, Athens, Greece.

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