3 Rhythm systems in music from across the world

THIS CHAPTER SPANS the music of several seemingly disparate countries and regions. What they have in common is that each has a unique approach to rhythm whose fundamental mathematical principles differ from those found in most Western music. It opens by introducing the concept of Euclidean rhythms; a family of cyclical rhythms whose structures are derived from the Euclidean algorithm and whose use can be found in traditional music across the world, not least in over 40 time lines. It then moves on to the time lines and rhythmic complexities of West African drumming. One of the underlying structural principles of Indonesian gamelan music, and indeed Balinese monkey chant, is its interlocking figurations. Much Afro-Cuban music also uses interlocking rhythms, here the rhythmic foundation is provided by the clavé, another Euclidean rhythm. The metric framework of *tal*, the repeating rhythm pattern found in North Indian classical music, is intrinsically linked to numbers and their complex manipulations, as too are the *palo* beat cycles of Spanish flamenco.

Euclidean rhythms

This section describes the way in which the structure of the Euclidean algorithm can be used to define a family of rhythms, known as 'Euclidean rhythms', many of which are found in traditional music across the world. The rhythms are cyclic: phrases or patterns that are repeated throughout a piece. Many of these are time lines that serve as rhythmic reference points in different musical traditions. The concept of the Euclidean rhythm in music was first written about by Godfried Toussaint in 2004 in his paper

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'The Euclidean Algorithm Generates Traditional Musical Rhythms' where he showed that the structure of the Euclidean algorithm may be used to generate a large family of rhythms used as time lines (ostinatos) in sub-Saharan African music in particular, and world music in general.[54]

The Euclidean algorithm for computing the greatest common divisor (GCD) of two integers is one of the oldest known algorithms (circa 300 B.C.). It was first described by Euclid in Proposition 2 of Book VII of *Elements*. It is a systematic repetitive procedure whereby two numbers are taken, the smaller number is subtracted from the larger one, and next the smaller number is subtracted from the difference then repeatedly replaced until both numbers are equal. This final number is then the GCD.

For example, consider the numbers 5 and 13. First, 13–5 = 8;

then 8 – 5 = 3;
next 5 – 3 = 2;
then 3 – 2 = 1;
and finally 2 – 1 = 1.

This means that the GCD of 5 and 13 is 1; in other words, 5 and 13 are relatively prime.

Toussaint discovered, through the work of Erik Bjorklund, an unexpected connection to timing systems in neutron accelerators; the same type of algorithm could be used to produce a binary string with a specified number of zeroes and ones. Bjorklund's algorithm, a version of the Euclidean algorithm, provided a means of visualising binary sequences which could also be considered in terms of a family of rhythms.[55] The GCD of two numbers was used by Toussaint to give the number of beats and silences in a rhythm; this generated many rhythms found in music across the world. Musical rhythms (and scales) can be seen as two-way infinite binary sequences. In a rhythm,

54 G.T. Toussaint, 'The Euclidean Algorithm Generates Traditional Musical Rhythms', Conference: In Proceedings of BRIDGES: Mathematical Connections in Art, Music and Science, 2004: 47–48.

55 For more details of Bjorklund's method see E. Bjorklund, 'A metric for measuring the evenness of timing system rep-rate patterns', SNS ASD Technical Note SNS-NOTE-CNTRL-100, Los Alamos National Laboratory, Los Alamos, USA, 2003.

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each bit represents one unit of time (for example, the length of a quaver or eighth note) and a zero bit represents a silence (for example, a quaver or eighth note rest) and the offset (or rotation) defines the starting pulse of a Euclidean rhythm.

In the Euclidean rhythm $E(k,n)$, k is the number of ones (onsets), and n (the number of pulses) is the length of the sequence (zeroes plus ones). So for example,

$E(5, 13) = [1001010010100]$.

The second representation is the box-like representation, also known as the Time Unit Box System, a sequence of n '×'s and ' · 's where '×' represents an onset and ' · ' denotes a silence. This notation is popular in the field of ethnomusicology. Using box notation, the rhythm above would be written as

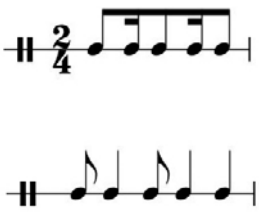
$E(5, 13) = [x..x.x..x.x..]$

This type of notation does not distinguish between the relative lengths of notes; rather it shows where the main notes of the rhythm, the onsets, appear. As Toussaint, and later others, discovered there are surprising numbers of Euclidean rhythms where *k and n* are relatively prime (numbers that are relatively prime have no common factors other than 1). Here are a few examples. The Euclidean rhythm $E(5, 8)$ can be found in the traditional music of Egypt, Korea, West Africa and Latin America.

$E(5, 8) = [× · ×× · ×× ·]$

It is the basis of, for example the Cuban *cinquillo* pattern.[56]

56 Cited in Erik D. Demaine, Francisco Gomez-Martin, Henk Meijer, David Rappaport, Perouz Taslakian, Godfried T. Toussaint, Terry Winogradf and David R. Wood. 'The distance geometry of music'. Computational Geometry 42 (2009): 429–454.



When it is started on the second onset it forms the rhythmic pattern of the Spanish tango.

Aksak rhythms

Euclidean rhythms are closely related to what are known as *aksak* rhythms. Aksak rhythms are commonly found in the rhythmic structure of the folk music of the Middle East and the Balkans, for example. They are characterized by combinations of those meters that use units of durations two and three, and no other durations. The Hungarian composer Bela Bartók referred to these as Bulgarian rhythms and used them frequently in his music (see pages 10-11). During the twentieth century *aksak* rhythms found their way into the works of a number of other twentieth-century composers, notably Stravinsky.

In 2004, the ethnomusicologist Simha Arom created an inventory of all the theoretically possible *aksak* rhythms for values of *n* ranging from 5 to 29 and listed all those that he found used in music across the world, these include the following.

E(2, 5) = [× · × · ·] = (23) (Greece, Macedonia, Namibia, Persia, Rwanda).
E(3, 7) = [× · × · × · ·] = (223) (Bulgaria, Greece, Sudan, Turkestan).
E(4, 11) = [× · · × · · × · · × ·] = (3332) (Southern India rhythm).
E(5, 11) = [× · × · × · × · × · ·] = (22223) (Bulgaria, Northern India, Serbia).
E(5, 13) = [× · · × · × · · × · × · ·] = (32323) (Macedonia).
E(6, 13) = [× · × · × · × · × · × · ·] = (222223) (Macedonia).
E(8, 17) = [× · × · × · × · × · × · × · × · ·] = (22222223) (Bulgaria).

E(8, 19) = [× · · × · × · × · · × · × · × · · × ·] = (32232232) (Bulgaria).
E(9, 23) = [× · · × · × · · × · × · · × · × · · × · × · ·] = (323232323) (Bulgaria)[57]

Euclidean rhythms can also be found in pop music. *E*(5, 8) is commonly found in rockabilly music and jazz, for example. In his article 'Kid Algebra: Radiohead's Euclidean and Maximally Even Rhythms', Brad Osborn identifies the use of Euclidean rhythms in various songs by the UK band. As he writes 'Radiohead uses Euclidean rhythms … that range from almost completely banal rotations of E(5,16) in 'Codex' to the jarring E(3,7) opening groove of '2+2=5' to the non-isochronous yet smooth E(4,10) recurring grooves in both 'Morning Bell' and '15 Step.'[58]

Rhythm in West African drumming

Africa is a huge continent with a wide variety of musical styles. The music played by West African drum ensembles is rhythmically very complex: multiple patterns are played at the same time creating many-layered rhythms which, in combination are built into complex and subtle constantly developing sound structures. Usually the music is not written down but passed on through oral tradition. Its complex rhythms are difficult to capture accurately in Western notation, partly because the concept of bars and the consequent strong and weak beats within the bar, is unknown in West African music. Although references are made to bars throughout this analysis, this is not a concept that would be perceived by the performers.

West African drumming uses a time line - a short repeated rhythm which is either clapped or played by a single or double bell. The performers follow the time line which effectively holds the piece together orienting time for the musicians and dancers as well as the audience. The African musicologist and composer Joseph Kwabena Nketia first used the term time line in 1963 describing it as 'a constant point of reference by which the phrase structure

57 Simha Arom. 2004. "L'aksak: Principes et typologie". *Cahiers de Musiques Traditionnelles* 17 (Formes musicales): 11–48.

58 Brad Osborn. 'Kid Algebra: Radiohead's Euclidean and Maximally Even Rhythms'. *Perspectives of New Music,* Vol. 52, N0. 1, (Winter, 2014): 81-105.

of a song as well as the linear metrical organisation of phrases are guided'[59] Here is one of the most common time lines found in sub-Saharan Africa, sometimes referred to by ethnomusicologists as the 'standard pattern'.

As a Euclidean rhythm it could be classified as

$E(7, 12) = [\times . . \times . . \times x . . x . . x . . x]$

The rhythmic structure of African drumming is divisive (rather than additive, see page 2); it often uses eight and twelve pulse patterns but these may be divided into both four groups of three and three groups of four, or into the irregular divisions of 7+5 or 5+7. Several different rhythms are played at the same time, and rhythm patterns interlock and overlap to form polyrhythmic patterns and cross-rhythms.[60] The rhythmic scheme of West African drumming could be described as polymetric where more than one metre is used at the same time. Philip Tagg refers to sub-Saharan rhythms as being 'composite' because 'two metres operate together as one'. The cross-rhythms are 'ongoing and continuous, not a temporary feature of the music'.[61] There are many different divisions of the twelve pulse pattern in 'Agbekor Dance' – the sogo part (the time line) is in groups of three whereas the atsimevu uses many different divisions.

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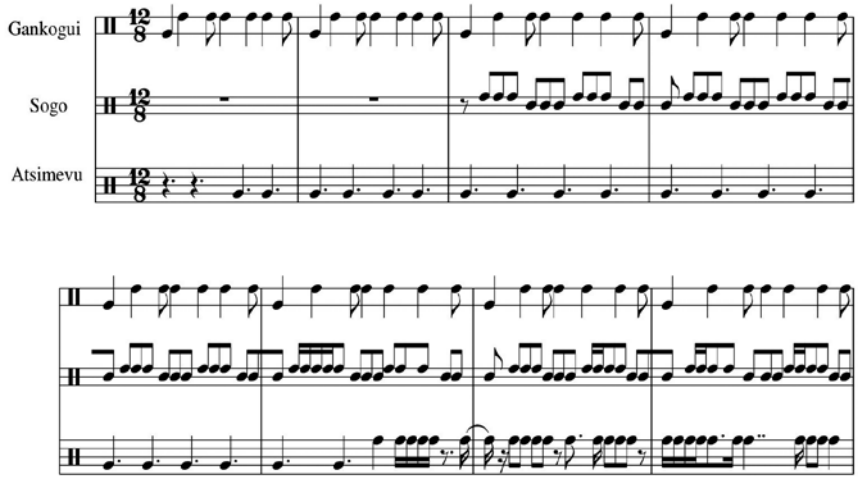
59 Joseph Kwabena Nketia. African Music in Ghana. Evanston, IL: Northwestern University Press, 1963.

60 Polyrhythms occur when two or more different types of rhythm are heard together against the same pulse, such as one drum in triple time playing against another in quadruple time. A cross rhythm is the effect produced when two conflicting rhythms are heard together using different metres. The terms polyrhythm and cross-rhythm are often confused.

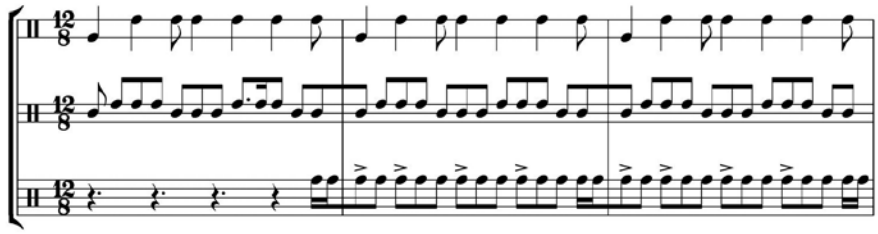
61 Philip Tagg. *Music's Meanings a modern musicology for non-musos* (New York and Huddersfield: The Mass Media Music Scholars 'Press, Inc., 2013): 457 citing J M Chernoff. *African rhythm and African sensibility, aesthetics, and social action In African musician idioms*. (Chicago and London: The University of Chicago Press, 1979).

The examples used below are taken from 'Agbekor Dance', a traditional Ewe battle dance from Ghana as recorded by the master drummer and ethnomusicologist Mustapha Tettey Addy. There are three instruments playing in the fast dance: the gankogui (bell) which plays the time line; the sogo drum; and the atsimevu played by the master drummer. The gankogui opens the playing with the time line which continues throughout the performance. The master drum then opens with a simple rhythm followed by the sogo whose entry creates a complex three-part polyrhythmic texture (see below).

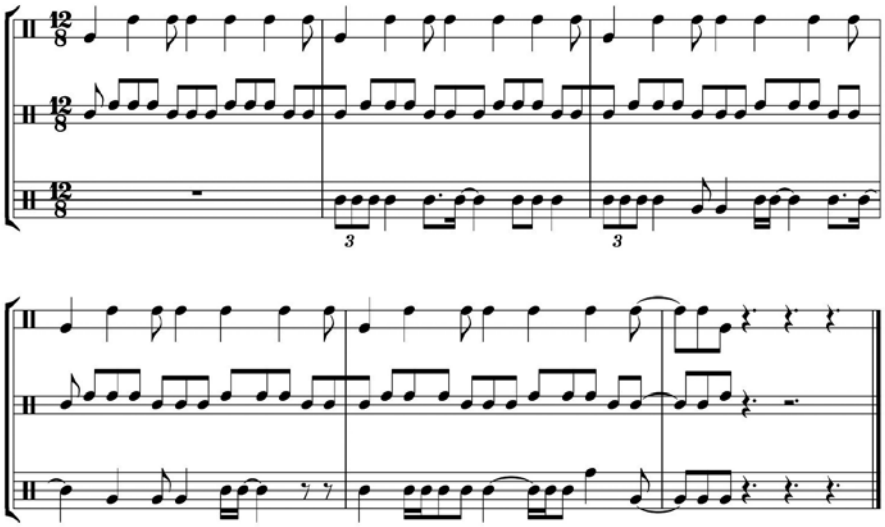


The master drummer has the most elaborate part in a drum ensemble and often plays solos. Part of their role is to lead the drum ensemble by giving musical cues in the form of rhythm patterns. As one of the pioneers of African music studies, A M Jones, wrote, 'The master drummer… has a number of standard patterns at his disposal …what he does having first established the pattern, is to play variations on it …besides being able to play the patterns at will, he can also play them in any order and need not play them at all.'[62] As the piece unfolds the patterns increase in complexity and virtuosity. Sometimes entries are staggered as below creating interesting cross rhythms. The parts operate in different metres and could be described as polymetric (see below).

62 A M Jones. *Studies in African Music*. (London: Oxford University Press, 1959): 80.



The rhythms get increasingly complex as the piece goes on as can be seen below in the final bars. The rhythms operate on three different strata each of the players using different figurations. The master drummer improvises complex figurations riding on the two levels above and with little use of repetition. As the battle dance closes all three players come together on the final three notes (see below).

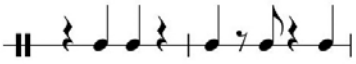


Much Afro-Cuban music also uses interlocking rhythms. Here the rhythmic foundation is provided by the clavé, the main rhythmic organizing principle, not only in music from Latin America but across the world in, for example, Central Africa, Cuba, the Middle East and the United States. In Spanish

clavé means key or keystone. The clavé rhythm is often played by a pair of small wooden sticks which are struck together and are known as claves. There are two versions of the standard clavé rhythm – forward (3 +2) and reverse (2 + 3). The commonly used clavé patterns are Euclidean rhythms which could be classified as E(5, 16) = [324222] in the forward rhythm and

E(5, 16) = [24334] (counting from the first onset) in the reverse version. Here is the forward clavé rhythm.

Here is the reverse clavé rhythm.



The clavé is similar in function to the West African time line in the way that it serves as a rhythmic reference point, holding the players together; its patterns determining how dancers move to the music.

The Indonesian gamelan

On first hearing live gamelan music in Bali, the Canadian composer and ethnomusicologist Colin McPhee (1900-1964) wrote of its interlocking rhythms:

> Gradually the music revealed itself as being composed, as it were, of different strata of sound. Over a slow and chantlike bass … the melody moved in the middle register, fluid, free, appearing and vanishing in the incessant shimmering arabesques that rang high in the treble as though beaten out a thousand little anvils. Gongs of different sizes punctuated this stream of sound, divided and subdivided it into sections and inner sections, giving it meter and meaning. Through all this came the rapid

> and ever-changing beat of the drums, throbbing softly or suddenly ringing out with sharp accents. They beat in perpetual cross rhythm …Tiny cymbals pointed up the rhythm of the drums, emphasized it with their delicate clash, while the smallest of bells trembled as they were shaken, adding a final glitter…[63]

Gamelan is the word used to describe an Indonesian ensemble made up mainly of tuned percussion instruments such as metal chimes and gongs of different sizes. A typical ensemble will usually have around 25 performers who work very closely together. Although there are a number of different gamelan traditions in Indonesia, they tend to share certain musical principles, such as polyphonic stratification, cyclical time structures and the complex construction of melodies that are shared seamlessly between two or more performers to produce a single strand of music. The basis of the music could be described as a core melody, a melodic outline around which other instruments perform closely interrelated variants at the same time creating an uninterrupted melody varying in complexity and density. In the words of the ethnomusicologist Leslie Tilley, 'One often cannot discern which musician has performed which note in an interlocking passage: the perceptual effect is of a group of musicians each playing the entire passage, in perfect synchrony, much faster than humanly possible'.[64] McPhee described the effect thus

> The brilliant, interlocking two-part figuration that is a unique feature in Balinese gamelan music is an ingenious and sophisticated elaboration of a primitive technique, in which a group of performers create rhythmic or melodic patterns by each man sounding in turn a single tone, so that a more or less unbroken continuity results.[65]

A form of this interlocking rhythmic device can be found in *katak* music, sometimes referred to as the Balinese monkey chant. Here a large chorus of

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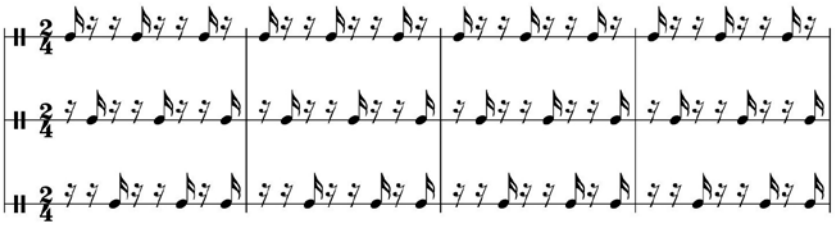
63 Colin McPhee. *A House in Bali* (Hong Kong: Periplus, 2002): 40-41.

64 Leslie Tilley. 'The draw of Balinese rhythm' in *The Cambridge Companion to Rhythm. Eds. Russell Hartenberger and Ryan McClelland.* (Cambridge: Cambridge University Press, 2020): 270.

65 Colin McPhee. 'The five tone gamelan music of Bali', *The Musical Quarterly*, Vol. 35, No. 2 (April, 1949): 272.

men, sometimes over 100, chant the syllable 'chak' in a syncopated pattern. One of the most common patterns used is known as *cak telu* a 3 + 3 + 2 rhythm made up of three notes (the rests are part of the count). Each of the three vocal lines enters a single note after the one before, creating a constant interlocking stream of notes which repeats many times (see below).



Gamelan has a colotomic structure where specified instruments are used to mark off established time intervals at numbered points.[66] Gongs of different sizes are used to mark the main divisions of cycles of music known as *gongan*. The Balinese, *bapang*, for example, is a common 8-beat structure where the medium-sized gongs (*kempur*, abbreviated as P) and small high-pitched gongs (*klentong* 't') form a symmetric pattern with *gong ageng* (G). They create a cyclic structure together as follows - G – P – t – P – G.[67] Thus the music is conceived in terms of articulated points rather than in terms of duration between points. Because the largest gong provides accents at the ends of gongan, this means that the music is end-weighted, rather than having accents at the beginning of bars and phrases as in much western music. Other examples of colotomic structure can be found in the *gagaku* (court music of Japan) where divisions are marked by a drum and hanging gong, and in the pi phat (percussion and wind instrument) ensembles of Thailand.

Rhythm in North Indian classical music

Rhythm is highly developed in music across the Indian subcontinent whether in the classical music of the North (Hindustani music), the South

66 In a colotomic structure, specific instruments (such as gongs) mark the beginnings and ends of rhythmic cycles.

67 Tilley, *Cambridge Companion*, 263.

(Carnatic music), or devotional or popular genres. This section focuses on North Indian classical music, the Indian musical tradition most familiar in the West. Other forms of Indian music, such as film music and Qawwali, borrow heavily from this tradition in terms of rhythm as well as form and instrumentation.

North Indian classical music ensembles have only a handful of players and most instruments are played while seated on the floor. The group will usually have the same three elements: a soloist, either a singer or an instrumentalist performing the melody; percussion, usually tabla; and a drone, either a harmonium or a tanpura. *Raga* and *tala* are the basic structural principles of Indian classical music essentially a linear melody over a cyclical rhythm. A *raga* (also spelled as *rag* in Northern India) is a pattern of notes that forms the melodic basis of an entire piece. Each *rag* has a particular ascending and descending pattern and is associated with a different time of the day, a season, particular mood or a special occasion.

The *tal* is a repeating rhythm pattern, usually played by the tabla (small drums). *Tal* can be traced back through musicological treatises over a period of approximately 2,000 years of continuous development. The South Indian or Carnatic tradition has its own *tal* system, which diverged from that of the North around 300-400 years ago.[68] The concept of *tal* is intrinsically linked to numbers. *Tal* are usually between six and sixteen beats long, although some are much longer. They have names and distinctive characteristics e.g. their length and specific divisions into subsections. The recurring patterns of beats are manifested in hand claps, waves, finger counts and drum strokes. In Hindustani music about 20 *tals* are commonly used, the most prevalent being *tintal* – four beats, each beat lasting for four counts.[69] The repetitive beat patterns are cyclic with smaller cyclic patterns embedded within them. They differ from typical Western music in that they do not include regularly recurring strong and weak beat functions, rather they are arranged in an abstract hierarchy according to whether they are indicated with a clap, a wave or a finger count. As such, *tal* is based on additive rhythm principles rather than the divisive principles used in Western music.

The metric framework of *tal* defines the rhythmic structure of the music. The beats are grouped into small sections within the pattern. So, for instance,

68 Martin Clayton. *Time in Indian music: Rhythm, Meter and Form in North Indian rag.* (Oxford: Oxford University Press, 2000): 11.

69 Clayton, *Time in Indian music*, 43.

tintal has sixteen beats (4 + 4 + 4 + 4) but this should not be thought of as the equivalent of 4/4. The beginning of the first, second and fourth sections is marked by a clap, but the beginning of the third section is weaker and this is shown by a wave of the hand. The sections are known as *vibhag* and the beats are known as *matras*. The tabla uses different strokes (*bols*) to mark the different beats. The framework of the stroke patterns is known as *theka*. Each *tal* has a particular *theka*. X marks the *sum* – the first beat of the cycle. 0 indicates silence and is usually marked by the wave of the hand with the palm turned upwards. These points are known as *khali vibhag* and are important reference points.

```
X               2               0               3
> clap          > clap                          > clap
 1  2  3  4  5  6  7  8  9  10 11 12 13 14 15 16
```

Every *tal* has a different identity. There are different ways in which the same number of beats can be grouped, as can be seen in the 14-beat *tals*, *jumhra* and *dhamar* below.[70]

Jumhra

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X           2           0           3
>           >           >
1  2  3  4  5  6  7  8  9  10 11 12 13 14
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=

Dhamar

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X                 2           0           3
>                 >           >
1  2  3  4  5  6  7  8  9  10 11 12 13 14
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70 Gerry Farrell. *Indian Music in Education*. (Cambridge: Cambridge University Press, 1990): 29-35.

There can be more than one *khali* in a *tal* as in *sultal* below.

Sultal

X		0		2		3		0	
>				>		>		>	
1	2	3	4	5	6	7	8	9	10

The *khali* may also occur on sum as in *rupak* below.

Rupak

X (0)			2		3	
			>		>	
1	2	3	4	5	6	7

Bols imitate the various stroke patterns on the tabla, their meaningless syllables are used to help memorise compositions. They can also be used as a means of notation. The *theka* for tintal is as follows.

X				2				0				3			
>				>								>			
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
dha	dhin	dhin	dha	dha	dhin	dhin	dha	dha	tin	tin	ta	ta	dhin	dhin	dha

Each tal has a particular *theka*.

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Jumhra

X			2				0			3				
>			>							>				
1	2	3	4	5	6	7	8	9	10	11	12	13	14	
dhin	dha	tirakitja	dhin	dhin	dhage	tirakitja	tin	ta	tirakitja	dhin	dhin	tirakitja	dhin	

Rupak

X (0)			2		3	
			>		>	
1	2	3	4	5	6	7
tin	tin	na	dhin	na	dhin	na

The tabla player is therefore manipulating several layers of metrical information: the basic outline of the *tal*; the framework of stroke patterns in the *bols*; and the distribution of the different strokes across the different *vibhag*. Numerous variations are possible within this framework and part of the table player's skill is in the ability to manipulate the rhythm within this. The basic structure of the *tal* remains intact but a musician can choose to disrupt these smaller patterns by contradicting them, eventually bringing the music back to the fundamental pattern of cyclical beats, usually on *sum*, the first beat of the cycle. *Sum* marks the beginnings and ends of improvisations so it is often accented. Some improvisations end with a thrice repeated short rhythmic pattern.

The rhythmic structure of Spanish flamenco

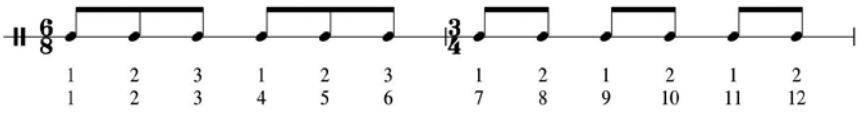
Just as North Indian classical music incorporates the hands into its rhythmic structure (in the form of hand claps, waves, and finger counts) so too does Spanish flamenco, a musical tradition in which hand claps are integral to the form. Flamenco is the generic term applied to a particular ensemble art form, mostly coming from Southern Spain, that utilises song (*cante*), dance (*baile*)

and guitar music (*toque*). The musical accompaniment is provided by guitar along with a rhythmical backbone where hand-clapping, stomping, and finger snapping are important elements. The flamenco *cante* uses a rhythmic structure in which the accented beats are distributed in a particular pattern in a short sequence that repeats at regular intervals. There are two kinds of *palmas* (hand clapping) used in accompaniment.

> Flamenco *palmas sordas* ('deaf *palmas*') have a dull sound and are made by cupping the palms at right angles to each other; they are used in quieter sections, during singing or melodic playing... *Palmas fuertes/ secas/claras* ('strong/ dry/clear *palmas*') have a sharp, crisp sound and are made with three fingers in the tensed palm of the other hand with both hands roughly aligned in the same direction; they are used during loud stomping, during the faster parts of accelerations... [71]

Flamenco pieces are usually in *palo* form, the most common *palos* being *bulerías*, *alegrías* and *soleá*. Each *palo* is distinguished by its basic metrical form and rhythmic structure (*compás*), along with its harmony and mood. A flamenco performance will usually contain several different *palos*. The compás is marked through various techniques such as the guitarist strumming, picking and tapping the soundboard, hand clapping (*palmas*) and finger snapping (*palillos*).

The most common beat cycle for *palos* is 12. The 12 beats are usually formed from a mixture of 6/8 + 3/4 bars. A common pattern used has bars of 6/8 followed by 3/4 which results in alternating groups of two and three beats.



71 Mariana Maduell and Alan M Wing. 'The dynamics of ensemble: the case for flamenco'. *Psychology of Music* Vol 35(4): 591-627.

However, the accents do not correspond to the downbeats as found at the beginning of bars in Western classical music. Notes are accented at different points and it is the placing of these accents which differentiate the *palos*. The lively 12-beat *palo Alegrías* originated in Cadiz. Its beat pattern is as follows, the accented notes are shown in bold:

1 2 **3** 4 5 **6** 7 **8** 9 **10** 11 **12**

The divisions corresponds with the **1** 2 3 **1** 2 3 **1** 2 **1** 2 **1** 2 pattern above but the accents do not fall on the downbeats.

One of the most popular *palos* is the bulerias, a fast flamenco rhythm made up of a 12 beat cycle where the emphases are placed at certain points in the two general forms shown below. Notice how, in both cases, the pattern begins on the 12th note of the cycle.

12 1 2 **3** 4 5 **6** 7 **8** 9 **10** 11

12 1 2 **3** 4 5 6 **7** **8** 9 **10** 11

The accompanying *palmas* are played in groups of six beats resulting in numerous counter-rhythms.

As has been illustrated, flamenco performances are closely tied to a strict rhythmic structure, but, in common with North Indian classical music, improvisation takes place within this and is seen as a measure of the performer's skill.

This chapter has illustrated a number of the rhythm systems found in music across the world. It has pointed out the differences between their foundations and those of Western music; a divisive system where bars and barlines are fundamental to the metrical structure and time signatures divide the beats, usually into groups of two, three and four with an accent on the first beat. In contrast, most of the music in the systems explored is not divided into bars, and time signatures (and indeed staff notation) are unknown. The rhythms of Western music are relatively simple in comparison with the given examples of music from around the world: the irregular divisions of aksak rhythms; the *tala* of North Indian classical music; the

interlocking figurations of gamelan; and the polyrhythmic patterns of West African drumming. Some systems are additive rather than divisive (North Indian music for example) and numbers, although featuring prominently in several styles, perform different functions: they form the basis of Euclidean rhythms; they define the *matras* and *thekas* of different *tals*; they are central to the colotomic structure of Indonesian gamelan; and they define the beat cycles of Spanish flamenco. Furthermore, unlike in most Western music, improvisation is integral to several of the musical styles described.

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