

5 Frieze patterns in music

FRIEZE PATTERNS ARE repeating patterns of elements arranged along a line or in a strip. They have existed for centuries as decorative adornments and occur frequently in architecture and decorative art. They often appear as borders; the term 'frieze' comes from architecture where it is used to refer to a decorative carving or pattern running horizontally just below a roofline or ceiling.

In mathematical terms a frieze pattern can be described as a plane figure with a symmetric pattern bounded by two infinite parallel lines. The patterns are repetitive in one direction according to the symmetries of the pattern. Frieze patterns can be classified in terms of their groups of symmetries, hence the mathematical study of frieze patterns focuses on group theory. Group theory is a branch of abstract algebra, which looks at the main features of a group, from both the point of view of its elements and its group operations. Symmetrical patterns are classified according to the types of symmetry which are used. There are exactly seven different frieze symmetry types. All frieze patterns have translation symmetry i.e. you can slide a horizontal frieze pattern along a fixed distance (to the left or right, up or down) and it will appear unchanged. As well as translational symmetry, the other six types also have different combinations of rotational, horizontal, vertical and glide symmetry. This chapter looks at the way that the symmetries of the seven different frieze patterns can be found in music.

Here is a list of the frieze patterns along with the International Notation.

Frieze pattern	Description	International notation
TYPE 1	Translation.	p111
TYPE 2	Translation and glide symmetry.	p1a1
TYPE 3	Translation and reflection along the horizontal line.	p1m1
TYPE 4	Translation and reflection along the vertical line.	pm11
TYPE 5	Translation and 180° rotation.	p112
TYPE 6	Translation, 180° rotation, horizontal and vertical symmetry.	pmm2
TYPE 7	Translation, 180° rotation, glide, and reflection along the vertical line.	pma2

Guide to the international notation. The second symbol refers to reflection along the vertical line – where this exists the letter 'm' is used, otherwise there is a 1. The third symbol refers to reflection along the horizontal line – where this exists the letter 'm' is used, otherwise there is a 1. If there is a letter 'a', this means that there is glide symmetry (where the figure is reflected (inverted) and then translated by being shifted horizontally). Where the fourth symbol is 2 this means that there is rotational symmetry, otherwise there is a 1.

In the 1970s, the mathematician John Conway created names relating to footsteps for each of the frieze groups. The footsteps represent the following symmetries.



These can be seen in the table below along with a musical illustration of each frieze pattern. In each case a simple musical motif has been used to illustrate the symmetric pattern.

The seven frieze patterns with musical illustrations

HOP translation.



TYPE 1 p111



STEP translational and glide symmetry.



TYPE 2 p1a1



JUMP translation and reflection along the horizontal line.



TYPE 3 p1m1



SIDLE translation and reflection along the vertical line.



TYPE 4 pm11



SPINNING HOP translation and 180° rotation.



TYPE 5 p112



SPINNING JUMP translation, 180° rotation, horizontal and vertical symmetry



TYPE 6 pmm2



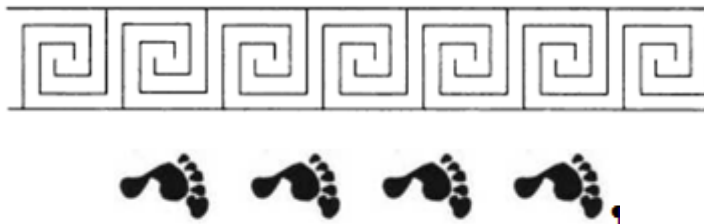
SPINNING SIDLE translation, 180° rotation, glide, and reflection along the vertical line



TYPE 7 pma2



TYPE 1 HOP Translation



The Type 1 frieze pattern is the most common. It is based on translation in a horizontal direction - it is simple repetition. Repetition is the easiest symmetry for the listener to perceive and as listeners we are often attracted to repetition; repeated ideas become familiar, they set up expectations, allowing us to anticipate and thus to reap the reward. As Arnold Schoenberg (1874-1951) wrote in *Fundamentals of Musical Composition* (1967): 'Intelligibility in music seems to be impossible without repetition.'⁸¹ The repetition of ideas helps bind music together. One idea is not enough, but if there are too many ideas following each other with no unifying factor then the music may sound shapeless and without a sense of direction.

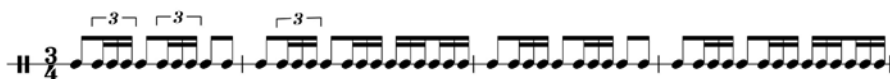
Schoenberg went on to write 'While repetition without variation can easily produce monotony, juxtaposition of distantly related elements can easily degenerate into nonsense, especially if unifying elements are omitted'. The French composer Erik Satie (1866 – 1925) risked potential monotony with his 1893 piece *Vexations*. The music is only half a page long and the piece bears the inscription *In order to play the theme 840 times in succession, it would be advisable to prepare oneself beforehand, and in the deepest silence, by serious immobilities*. It was 70 years before anybody took up the challenge of a 'full' performance.⁸² In 1963, the American composer John Cage (1912-1992) organised a team of players for the first performance at the Pocket Theater

81 Arnold Schoenberg. *Fundamentals of Musical Composition* (London: Faber & Faber, 1982): 103.

82 In his study of performances of *Vexations*, Gavin Bryars writes that the first performance of the piece that he knows of was given in 1958 by Richard David Hames at his school when he was 13 years old. For a list of most of the performances of this work up to 1979, see Gavin Bryars, "Vexations and its Performers," *Contact* 26 (1983): 12-20.

stays in the same form but eventually improvised licks are heard around it until the piece descends into exuberant chaos and the ostinato briefly collapses. For a few bars it is no longer there, but it is still firmly imprinted on the mind. When, after a few bars, it returns it is more a disfigured shadow of itself.

A frequently played piece that uses much repetition is the one-movement orchestral work *Bolero* by Maurice Ravel (1875-1937). The opening two-bar snare drum ostinato is repeated throughout the 17-minute work (see below). An ostinato is a short musical pattern repeated throughout a section or complete piece. In *Bolero* two alternating melodies are passed around the instruments above the relentless rhythm.



Before its first performance Ravel 'issued a warning' to the effect that 'the piece was an exercise in orchestration with 'no contrasts, and practically no invention except the plan and the manner of execution'.⁸⁶ The interest lies in the brilliant orchestration, the hypnotic power of the unflagging rhythm, and the steady crescendo which leads to an exciting climax. The piece was an instant success with audiences. Ravel, however, once described it wryly to his friend the composer Arthur Honegger as 'a masterpiece...without any music in it'.⁸⁷

There are, loosely speaking, two types of repetition used in music. These are the repetition of short units (a few bars, a theme or a motif, for example) and repetition of large sections of a piece of music as a structural device (in binary, ternary and sonata form, for example). Repetition allows themes to be heard again and for patterns to be displayed.

The Italian composer Domenico Scarlatti (1685 – 1757) wrote over 500 keyboard sonatas, originally intended to be played on harpsichord or fortepiano. Virtually all the pieces are written in simple binary form. However, they are abundant with musical ideas, interesting rhythms and harmonies, remote keys (for the time) and unprepared modulations. One

86 *Daily Telegraph*, July 11, 1931.

87 A. Honegger, *Incantation aux fossils* (Lausanne: Editions d'Ouchy Lausann, 1948): 91–2.

of the musical characteristics found in many of the sonatas is the use of repetitions of the same musical phrase, one after the other – as in the Type 1 frieze pattern. Here is an example taken from his Keyboard Sonata K 27 (bars 11-16) where the same two bars are played three times consecutively.

A commonly found use of repetition popular in Baroque music is the ground. A ground is a recurring melody, usually in the bass, accompanied by continuous variation in the upper parts. The term ‘ground’ may refer to the bass line, or to the entire musical scheme including the harmonies and upper voices. A ground bass is a repeating bass line which forms the basis of a set of continuous melodic and/or harmonic variations. The bass line repeats many times whilst the variations usually get more and more complex as the piece goes on. It was particularly popular during the Baroque period in music and often formed the basis for the passacaglia and the chaconne.⁸⁸ The bass line of the famous *Canon for three violins and continuo* by Johanne Pachelbel (1653-1706) has only eight notes forming a two-bar phrase (see below). This is repeated throughout the piece – 28 times in all.

88 The passacaglia is typically in triple time, in a minor key with a four-note pattern descending by step. The chaconne, although usually minor, sometimes uses a major key.

is heard four times each time the repetition is varied with changes in texture and instrumentation.

The work of Minimalist composers such as Steve Reich (b 1936), Philip Glass (b 1937) and John Adams (b 1947) features constantly repeated patterns that are subjected to gradual processes of melodic and rhythmic change (transformation) as the piece unfolds. Such transformations include phase shifting (or phasing) where one part repeats constantly and another gradually shifts out of phase with it (see pages 30-4).

Pop music in general makes much use of repeated chord progressions, drum patterns, and riffs not mention whole sections, such as the verse and chorus. Many pop songs are built on riffs - short, repeated melodic or chordal patterns which may be heard at different pitches. Riffs first came to prominence in jazz, swing and rhythm and blues. They formed the basis of the song's structure where musical variants took place above them. In his definition of riffs, the pop musicologist Richard Middleton notes their structural importance, describing them as 'short rhythmic, melodic, or harmonic figures repeated to form a structural framework'.⁹¹ Certain riffs are typical of the blues style. The pattern below, for example, would be repeated through the song with the same shape being built on the different chords of the 12-bar blues.



Amongst songs including the best-known riffs are Led Zeppelin's 'Whole lotta love' (1969), 'Sweet Child O' Mine' (1987) by Guns 'N' Roses and 'Seven Nation Army' by the White Stripes (2003). What they have in common is that they are both memorable and catchy, so much so that the 'Seven Nation Army' riff with its roots in garage blues was transformed into a political anthem sung to the words 'Oh, Jeremy Corbyn' when he was leader of the UK Labour party. Thousands joined in with the chant at the Glastonbury Festival in 2017.

In 1977 Donna Summer and Giorgio Moroder collaborated on the disco song 'I feel love'. Combining Moroder's incessant synthesizer bass-line, swirling

91 Middleton, Richard. *Studying Popular Music*. (Philadelphia: Open University Press, 1990)..

chords and programmed drums with Summer's ecstatic vocal floating over it, the song was to serve as a blueprint for house and techno, two of the earliest forms of Electronic Dance Music (EDM).⁹² A simple one-bar pattern in the synthesizer is repeated over and over, for nearly six minutes (roughly 350 repetitions), helping to create a trance-like state. Here is the synth bass line.



following bell pattern (see below). Its role is vital in maintaining the flow, energy and clarity of the entire ensemble. L and S refer to long and short strokes (see pages 41-2 for more detail).



TYPE 2 STEP Translation and glide symmetry



The Type 2 frieze pattern uses glide symmetry which means that the figure is reflected (inverted) and then translated by being shifted horizontally to the right.



The next two examples, Types 2 and 3, are both taken from Igor Stravinsky's score *Petrushka*. *Petrushka* is a ballet in four scenes which tells the story of three puppets who are brought to life. It is scored for large orchestra with a prominent piano part and was first performed by the impresario Diaghilev's *Ballets Russes* in Paris in 1911.

The following extract is taken from the first scene in the section titled 'Russian Dance'. For ease of reading, the music below has been written using treble clefs. The top two staves are the Bb clarinets and the lower two staves are the viola parts. In both instruments, the music on the second beat is an inversion of the rising arpeggio figure on the first beat. The Type 2 frieze pattern uses glide symmetry which means that the arpeggio figure is reflected (inverted) and then translated by being shifted horizontally to the right.



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There are many other examples of uses of musical symmetry in this score. Did Stravinsky (1882-1971) make a conscious decision to use such musical devices? We know from his writings that the answer is almost certainly yes. As well as being a composer, Stravinsky was an influential thinker about music and he wrote much on the subject. His work *Poetics of Music in the Form of Six Lessons* (1942) is considered to be a landmark of twentieth century musical aesthetics. In this he writes at length about his own compositional process.

The creator's function is to sift the elements he receives from her, for human activity must impose limits upon itself. The more art is controlled, limited, worked over, the more it is free.⁹³

TYPE 3 JUMP Translation and reflection along the horizontal line



93 Igor Stravinsky. *Poetics of Music in the Form of Six Lessons*. (London: Harvard University Press, 1942).

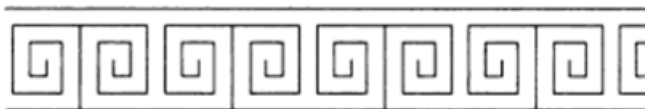


The Type 3 frieze pattern uses translation and reflection along the horizontal line. This means that the descending scale figure in the example below is reflected along the horizontal line (inverted). The bars of music below are taken from the harp part of ‘The Shrove – tide fair’.



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TYPE 4 SIDLE Translation and reflection along the vertical line



The Type 4 frieze pattern uses translation and reflection along the vertical line. This means that in each pair of footprints the motif is reflected along the vertical line (played backwards).

The following examples illustrate the ways in which two composers from different centuries have used this type of frieze pattern in their keyboard music. The first example is from the eighteenth century where Domenico Scarlatti uses the device in his Keyboard Sonata K. 514 in C (see below).



Notice how the second half of the bar in the left hand is a retrograde (backwards) version of the first half of the bar in the right hand.

The second example is taken from the twentieth century. The way that the French composer Claude Debussy (1862–1918) wrote for piano altered the way in which the instrument was perceived; he often used the extreme registers, with widely spaced chords and parallel movement as well as taking a new approach to pedalling which produced subtle blending effects. *Estampes* (1903) is a set of three pieces for piano. The first piece *Pagodes* evokes the Javanese gamelan which Debussy had heard played at the 1889 Exposition Universelle in Paris (for bars 26 – 29, see below).

The image shows a musical score for Claude Debussy's *Pagodes*. It is in 4/4 time with a key signature of three sharps (F#, C#, G#). The right hand (treble clef) features a complex texture with triplets of eighth notes. The first half of the bar contains a triplet of eighth notes, followed by a quarter note, and then another triplet of eighth notes. The second half of the bar is a retrograde version of the first half, with the first quarter of the second triplet acting as the point of reflection. The left hand (bass clef) plays a simple accompaniment of quarter notes. The score includes a *pp* dynamic marking and a pedalling instruction (ped.) with a line under the first two measures.

In this example above, the Type 4 frieze pattern is found in the right hand: the second half of the bar is a retrograde version of the first half with the first quarter of the third triplet acting as the point of reflection.

TYPE 5 SPINNING HOP Translation and 180° rotation



The Type 5 frieze pattern combines translation with 180° rotation (retrograde inversion). The original musical idea is combined with the retrograde inversion. Béla Bartók (1881-1945) uses the Spinning Hop frieze pattern in the fifth and final movement (Allegro molto) of his *String Quartet No. IV* (for bars 329-331, see below).

Here is the first violin part with its 180° rotation immediately underneath it.



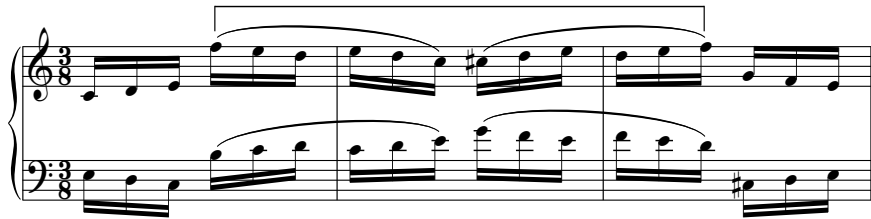
The rotation is played by the viola in the first bar of the example and by the cello in the second and third bars.

TYPE 6 SPINNING JUMP Translation 180° rotation, horizontal and vertical symmetry



The Type 6 frieze pattern is one of the most complex in its construction. It combines translation with 180° rotation along with reflections about both the x and y axis. The upper stave is formed by alternating the original with a retrograde of the original. The lower stave is an inversion of the upper stave.

Musical examples of Type 6 are very difficult to spot. The example below from Beethoven's *Diabelli Variations* is almost, but not quite, there. Conway's footprints shown below correspond with the middle complete bar of the Beethoven in terms of the overall shape. The original motif in Variation XXVI is followed by the retrograde version in both staves, and the lower stave is an inversion of the upper stave (bars 9-11). However, strictly speaking the notes on the second beat in the R H should be C D E rather than C# D E. Similarly the C D E of the L H should be followed by E D C (rather than G F E).



TYPE 7 SPINNING SIDLE Translation 180° rotation, glide, and reflection along the vertical line



The Type 7 frieze pattern is similar to Type 4 in that it uses translation and reflection along the vertical line. This means that in each pair of footprints the motif is reflected along the vertical line (played backwards). In Type 7 this is combined with 180° rotation and glide as can be seen in the middle pair of footprints.



The example below is taken from a keyboard suite by Handel. This is the Prelude, the opening movement of *Suite No. 1* marked *quasi fantasia*. Such improvisatory flourishes were typical of Baroque keyboard music particularly in the opening bars of toccatas and preludes.



This chapter has shown how the symmetries of each of the seven frieze patterns can be found in music. Chapter 9 looks in more detail at the ways in which symmetries are fundamental to the musical structure of two musical forms in particular, the canon and the fugue.