7 More scales: whole tone, octatonic, pentatonic, and scales that use microtones

THIS CHAPTER OPENS by describing modes of limited transposition, such as the whole tone scale and the octatonic scale, giving examples of their use in Western classical and pop music. It then moves on to the ubiquitous pentatonic scale, the five-note scale which is found in different forms across the world. The next section looks at some of the scales formed by dividing the octave into more than 12 semitones, made possible through the use of microtones, intervals smaller than a semitone. It discusses the long history of experimentation with equal temperament systems, notably the 19-note, 31-note and 53-note scales, and describes some of the instruments devised to accommodate them. The chapter concludes by looking at two twentieth-century composers who have used microtonal material in their music; the Czech composer Alois Hába was one of the first and went on to establish a department of microtonal music in Prague. Sometime itinerant hobo, the American Harry Partch, was similarly devoted to the cause of microtonal music, he rejected Western scales and invented his own, one of which was made up of 43 notes to the octave.

Modes of limited transposition

Olivier Messiaen was the first to use the term modes of limited transposition to denote scales which can be transposed a limited number of times before the original set of pitches reappears. The symmetry inherent in these modes means that no note can be perceived as the tonic they lend themselves to static harmony or what Messiaen described as containing "the charm of impossibilities".[102]

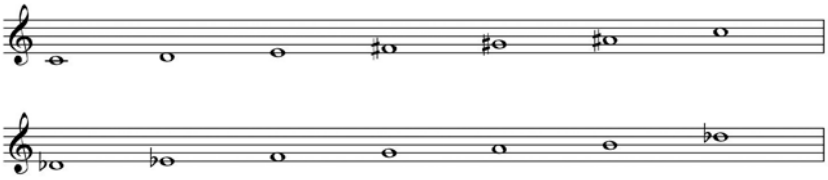
102 Messiaen, Olivier. *Technique of My Musical Language* (Paris: Alphonse Leduc, 1944): 58.

10.5920/mathsandmusic.7

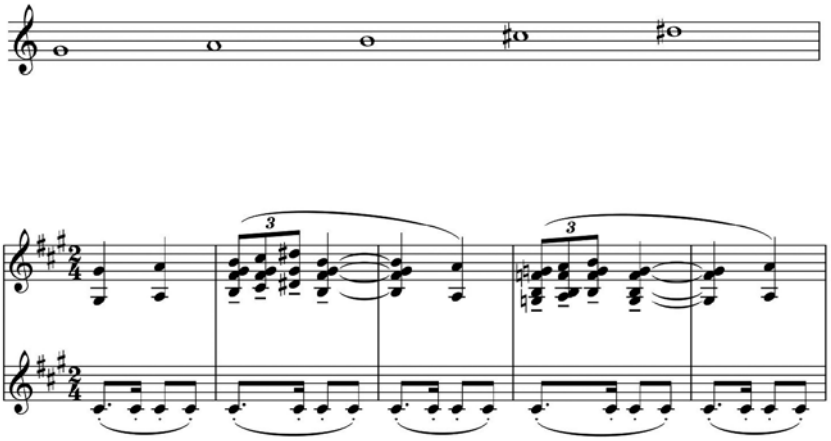
The two most common examples of modes of limited transposition are the whole tone scale and the octatonic scale (both of which were already in existence before Messiaen coined the term).

Whole tone scale

The whole tone scale is made up of six whole tones starting on either C or Db (its only transposition).



Messiaen refers to this as his first mode of limited transposition. Another French composer, Claude Debussy (1862-1918), uses the whole tone scale in 'La soirée dans Grenade' from his piano suite *Estampes*. In this languid movement, Debussy evokes images of Spain through strumming guitar sounds and a *habanera* rhythm. From bar 23 onwards he uses notes from the whole tone scale below to evoke the exotic atmosphere.

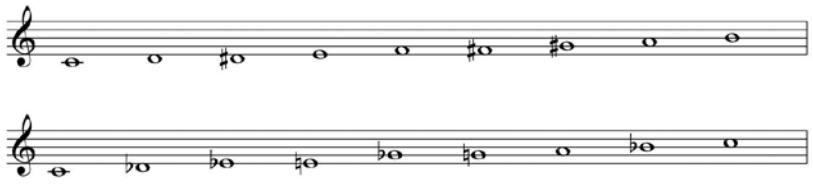


This corresponds with the second whole tone scale shown above, but using enharmonic equivalents for Db (C#) and Eb (D#).[103]

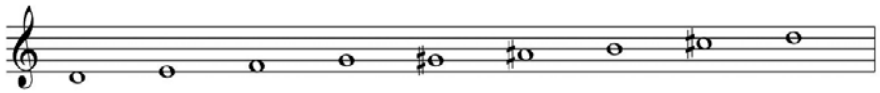
The whole tone scale is not often found in pop music but Stevie Wonder uses the scale in the Intro to 'You are the sunshine of my life' and it is found relatively frequently in jazz, by performers including John Coltrane, Thelonius Monk and Art Tatum.

Octatonic scale

The octatonic scale is made up of eight notes alternating tones and semitones. Each octatonic scale has two versions or modes; one starting with a whole step (tone) and the other starting with a half step (semitone). So, for example, starting on C the two modes are



The octatonic scale can start on any note, but in all, because the same series of notes appears with different starting notes, it is limited to three different patterns. As well as the two above, the remaining pattern is as follows.



Messiaen refers to this as his second mode of limited transposition. Instances of the octatonic scale are generally fairly rare although, as Allen Forte notes

103 Notes that sound the same but are written (or 'spelt') differently are said to be enharmonic e.g. Db and C# where Db is the enharmonic equivalent of C#.

in his article about Debussy's use of the octatonic scale, '…it should be emphasised that '… octatonicism was "in the air" during the early twentieth century'. Along with works by Debussy (including the opera *Pelléas et Mélisande*), Forte goes on to cite Strauss's opera *Elektra* (1908), the second of Webern's *Four Pieces for Violin and Piano* (1910), and Ravel's song 'Surgi de la croupe et du bond' (1913) from the *Trois poèmes de Mallarmé*.[104] Scriabin's *Sixth Sonata, Op. 62* for piano (1911) could be added to this list as could its use in Stravinsky's ballets *Petrushka* and *Rite of Spring* and several pieces in Bartók's *Mikrokosmos* including 'From the island of Bali'. A rare example to be found in pop music is the intro to the Radiohead song 'Just'.

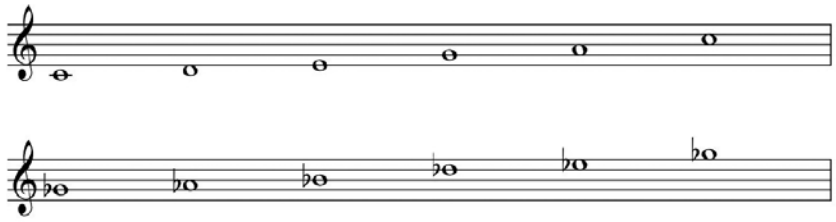
Pentatonic scales

All the scales discussed so far are predominantly found in Western classical music and most have seven notes. In contrast, the most widespread scales, both geographically and historically, are pentatonic scales formed of five pitches. They can be found in traditional music across the world from the British Isles, to China, Japan, Lapland, Hungary, West Africa and in the music of the indigenous people of the Americas. There are also examples in Western classical music through the centuries, from Gregorian chant to the piano music of Debussy and they can be heard in some blues, gospel and rock music. Three well-known examples of songs using pentatonic scales are 'Amazing grace', 'Nobody knows the trouble I've seen' and 'Auld Lang Syne'. Pentatonic scales are useful when improvising because they work well over several chords and have a major/minor ambiguity. They are not uncommon in pop music, 'Shape of you' by Ed Sheeran, is an example as are several songs by The Smiths including 'Heaven knows I'm miserable now' and 'There is a light that never goes out'.

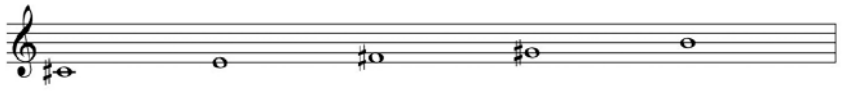
Pentatonic scales can be categorized as either hemitonic (with semitones) or anhemitonic (without semitones). The formation varies from country to country, and region to region. Two of the most well-known forms are found below. The first example uses only the white notes of the piano, whereas the second uses only black notes.

104 Allen Forte. 'Debussy and the Octatonic'. *Music Analysis*, Vol. 10, No. 1/2 (1991): 159.

Note that each of these contains the circle of 5ths. In the first example, for instance, C-G-D-A-E



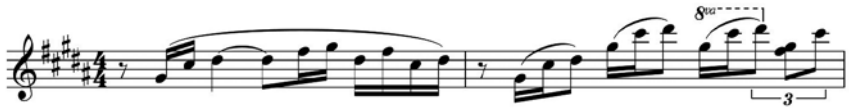
Slendro, an anhemitonic pentatonic scale, is one of the two major tunings in Javanese gamelan music, the other is pelog, a seven note scale . The five notes of slendro approximate those shown below.



At first sight the intervals employed in slendro seem to equate to the pentatonic scale on the black keys, a mixture of whole tones and minor thirds which are easily distinguished by ear. However, the five notes of slendro are more or less equally spaced, the resultant intervals lie somewhere between a whole tone and a minor third although this is not standardized and differs from one gamelan to another – the intervallic structure is known as *embat* and this marks out the personality of the gamelan.[105]

In 1889 Debussy attended the Exposition Universelle in Paris where he was particularly impressed by a Javanese gamelan with its ensemble of tuned percussion, gongs and metallophones of different sizes. In his evocative piece 'Pagodes', from the piano suite *Estampes*, he attempted to capture some of the Javanese gamelan sound partly through his use of the pentatonic scale which he uses to emulate the slendro scale. Here is the melody line in bars 3 - 4.

105 Neil Sorrell. *A Guide to the Gamelan*. (London: Faber and Faber, 1990): 56.



A preponderance of consonant intervals can be formed from the pentatonic scale. The nineteenth-century physician Hermann von Helmholtz (1821-1894) explained the relative consonance and dissonance of musical intervals in terms of the extent of the beating between the two notes when they are heard simultaneously. He discovered that notes whose fundamental frequencies are related by small whole number ratios have reduced beating because of the coincidences between the frequencies of the harmonics involved. Thus a perfect 5th, with a frequency ratio of 3:2, is a smooth/consonant interval with low beating because the 3rd harmonic of the lower note coincides with the 2nd harmonic of the upper note. In his study of the relative number of consonant and dissonant intervals formed by pairing notes in different scales, David Huron discovered that the pentatonic scale exhibited an optimum number of consonant intervals. He argued that the

> frequency of occurrence of various interval given pitch-class set can be correlated with corresponding perceived consonance for each interval class. If one of the composer's aesthetic goals is to generate predominantly consonant music, an appropriate choice of palette would maximize the availability of consonant harmonic intervals while minimizing the presence of dissonant harmonic interval'.[106]

It is difficult to give reasons for the ubiquity of the pentatonic scale, but it may be connected with the fact that it can be generated from the circle of 5ths and what has been described as optimum consonance.[107] Another reason may be its harmonic adaptability when improvising. However, the extent of the influence these features account for in terms of ubiquity is merely speculative.

106 David Huron. 'Interval-Class Content in Equally Tempered Pitch-Class Sets: Common Scales Exhibit Optimum Tonal Consonance' Source. *Music Perception: An Interdisciplinary Journal*, Spring, 1994, Vol. 11: 289-305.

107 Huron, 'Interval-Class Content', 301.

Scales that divide the octave into more than 12 notes

Most of the scales described so far divide the octave up into 12 semitones. The next section looks at some of the scales formed by dividing the octave into more than 12 divisions. This is made possible by the use of microtones, intervals smaller than a semitone. The term quarter tone is sometimes used, but this is not always accurate given that the interval may be larger or smaller than a quarter tone.[108] Since antiquity mathematicians and musicians have experimented with such equal temperament systems, notably the 19-note, 31-note and 53-note scales. The interval tunings that arise from these divisions are often closer to those found in the harmonic series (just intonation) rather than in 12 note equal temperament.[109]

19-note, 31-note and 53-note systems

Interest in the 19–note and 31-note systems dates from the sixteenth century when such systems were explored by various mathematicians. Könnicke's 'Harmonie-Hammerflügel' use of six diatonic manuals to divide the normal 12-key octave into 31 notes has already been mentioned (see page 108).[110] This was not the first instrument to experiment with this system. In 1637 Mersenne described a keyboard instrument utilising the 31-note scale in his treatise *Harmonie Universelle*, an outline of his theoretical and practical ideas on music. The instrument functions by having alternative keys for some notes, the tunings for these alternatives differing by the syntonic comma.[111] The Dutch mathematician and astronomer Christiaan Huygens (1629- 1695) took a different approach; he used logarithmic measurements of intervals to ascertain the equal division of the octave into 31 parts. His

108 Microtones are commonly used in contemporary near Eastern musical traditions such as those found in some Turkish, Arabic, Persian and Greek music.

109 In their study of microtones and projective planes, Wilson and Gamer compare the number of cents found in various intervals across scales using just ratios, 12 notes, 19 notes, 31 and 53 notes. See Robin Wilson and Carlton Gamer 'Microtones and projective planes' in John Fauvel, Raymond Flood, and Robin Wilson. *Music and Mathematics*. (Oxford: Oxford University Press, 2003): 148.

110 Michael Latcham. 'Könnicke, Johann Jakob' in *Grove Music*.

111 Neil Bibby. *Music and Mathematics. From Pythagoras to Fractals*. (Oxford: Oxford University Press, 2003): 22-23.

ideas are summarised in his *Lettre touchant le cycle harmonique* (1691) which includes a description of an imaginary transposing harpsichord, with 31 strings to the octave and a shifting keyboard of 12 notes which used various selections from the 31-note set.

Interest in the 53 note system can be traced back to antiquity. It originated from the fact that a series of 53 perfect 5ths equates very closely to 31 octaves. The difference between the two was calculated as $3^{53}/284$ by Nicholas Mercator (1620-1687) hence its name, the Mercator comma. Although conventional staff notation can be adapted for the 19-note and 31-note systems, this proves difficult with 53 notes, as does performance.

Indian classical music and the 22-note system

Indian music uses just intonation and the use of equal temperament is unknown. In Hindustani (North Indian) classical music, an octave is called *saptak* and has seven notes called *swara*. These can be further divided into 22 notes known as *sruti* or *shruti*. The seven notes are sa, re, ga, ma, pa, dha, ni (similar to the Western do re mi fa so la ti). The notes of the Indian scale are as follows.

Shadja (Sa), *Rishabha* (Re), *Gandhara* (Ga), *Madhyama* (Ma), *Panchama* (Pa), *Dhaivata* (Dha), *Nishada* (Ni).

The 22 note system was developed from early experiments in ancient and medieval times based on the fact that the highly concordant notes *panchama* and *madhyama* have ratios of 3:2 and 4:3 respectively. Further research was undertaken to determine the other ratios.

10.5920/mathsandmusic.7

Table 1 – Ratios used in the Indian scale

Degree of scale	Ratio
Sa	1
Re	9/8
Ga	5/4
Ma	4/3
Pa	3/2
Dha	5/3 or 27.16
Ni	15/8
Sa	2/1

The research examined both the cycle of 4ths and the circle of 5ths (see pages 104-5). It was found that the 12th note of the cycle in one case and the 11th and 12th notes in the other case were higher or lower by a small interval, again this gap was found to be in the ratio 81/80, equivalent to the 'syntonic comma' but known as the *praman* or *pramana sruti* in Indian music theory. Using *srutis*, precisely tuned microtones, the scale can be further divided into 22 notes.[112] The first and fifth notes (sa and pa) have only one variant, whereas all the other degrees of the scale (re, ga, ma, dha, and ni) have two variants each. There are ten pairs of notes and these along with the sa and pa give the 22 *srutis* of the Indian musical scale, these are mainly used in ornamentation. The two notes constituting each pair are found to be uniformly separated by the 81/80 interval of a comma or *pramana sruti*. The notes re, ga, dha, and ni have natural and flat variants, while ma has a natural and a sharp variant.

Microtonal material used in twentieth century classical music

During the twentieth century a number of Western classical composers incorporated microtonal material into some of their music, notable examples

112 P. Sambamurthy. *South Indian Music Book V. (Chennai: The Indian Music Publishing House, 1963).*

including Charles Ives 'Three Quarter-Tone Pieces' (1925) for two pianos tuned a quarter tone apart, and Karlheinz Stockhausen, whose electronic piece *Studie I (*1953) uses 25 equal-tempered intervals within each span of two and a half octaves but the most notable pioneers of microtonal music were perhaps Alois Hába and Harry Partch.

Alois Hába

The Czech composer Alois Hába (1893-1973) could be credited as the first to use quarter tones and sixth tones in Western art music. His parents were both folk musicians and his mother, a singer, taught him peasant songs from Moravia. These could be characterised by their use of microtones to heighten the expressive effect, the major mode using quarter-tone and sixth-tone sharps, the minor by quarter-tone flats. In 1917 he composed his Suite for string orchestra, his first composition to use quarter-tones and went on to write a great many microtonal works from piano pieces to an opera. In 1923 he established a department of microtonal music at the Prague Conservatory. In order to perform microtonal music Hába constructed special instruments: three types of quarter-tone piano, a quarter-tone and a sixth-tone harmonium, and a quarter-tone clarinet, trumpet and guitar. The opera *Matka* ('The Mother', 1929) is arguably Hába's masterpiece.[113]

Harry Partch, *U. S. Highball (A Musical Account of a Transcontinental Hobo Trip*)

The American composer, instrument maker and philosopher Harry Partch (1901 – 1974) rejected Western scales and invented his own, one of which was made up of 43 notes to the octave. His work was inspired by ancient Greek music theory and the work of Herman von Helmholtz.[114] Partch describes

113 Jiří Vysloužil. 'Hába, Alois' in *Grove Music*..

114 Herman von Helmholz. *On the Sensations of Tone as a Physiological Basis for the Theory of Music*, 1865 (London: Dover, 1954).

how he arrived at this scale and others in his book *Genesis of Music* (1949).[115] The scale is based on a 'tonality diamond'; an arrangement of ratios in a diamond shape where the diagonals sloping in one direction form 'Otonalities' and the diagonals in the other direction form 'Utonalities'. Partch's tonality diamond is similar to the diamond devised by Max Friedrich Meyer some 50 years earlier.[116] Having devised the 43-note scale, Partch devoted his time to implementing this system, creating new instruments that could be tuned to this new scale and then training performers to play them.

In 1935 at the height of the Great Depression he began a transient existence in the western states, jumping trains, sometimes living in boxcars in train yards, and picking up work occasionally. For several years he kept a small notepad that he used to transcribe fragments of conversations, writings on the sides of boxcars, names of stations and thoughts. These notes became the basis for *U.S. Highball*, a piece for speaker and Partch's instruments. In 1943 he started work on the first draft using the motley assortment of instruments he had built at that point. These included the Adapted Guitar (a guitar with new frets to accommodate his tuning system), the Chromelodeon (a re-tuned foot-pump harmonium), and the Kithara (his own version of the ancient Greek stringed instrument but now with 72 strings). New instruments were added for the second version including a set of marimba-based instruments; a smaller version of the 72 string Kithara (Surrogate Kithara), and a collection of tuned artillery shells and Pyrex water containers. Partch took the comments he had recorded in his notebook and turned them into melodies including quotes from hoboes such as "Hey, don't sleep with your head against the end of the car! You'll get your neck broke when she jerks!" With the small intervals in his new system of tuning applied to both voice and instruments, sliding steam whistle sounds and occasionally rhythmic percussive sounds, *U. S. Highball* has a surreal atmosphere, but it is oddly reminiscent of a train journey.

This chapter and the preceding one have outlined some of the mathematical and scientific thinking behind the creation and analysis of common and less common scales. This field encompasses a vast area of research which has

115 *Harry Partch. Genesis of a Music: an Account of a Creative Work, its Roots and its Fulfillments* (Madison: WI, 1949, enlarged 2/1974).

116 For more detail of the tonality diamond see Chris Forster. *Musical Mathematics. On the Art and Science of Acoustic Instruments*. (San Francisco: Chronicle Books, 2010).

10.5920/mathsandmusic.7

attracted many leading musicians, mathematicians and thinkers over the centuries including J S Bach and Messiaen, Pythagoras, Newton, Kepler, Mersenne, and Hertz, so that inevitably the surface has only been scratched both historically and geographically. In the twenty-first century, technological advances mean that electronic music facilitates any kind of tuning without the need for retuning or the construction of customised instruments.

10.5920/mathsandmusic.7