10 Proportion: Golden Section and the Fibonacci series

GOLDEN SECTION AND the associated Fibonacci series give rise to satisfying natural proportions. This chapter explores their use in music, looking in particular at the music of Mozart, Satie, Bartók, Debussy, and composers associated with the Darmstadt School, questioning whether such ratios are used consciously by composers or otherwise. The final section outlines the theory of proportional parallelism in the music of J S Bach. It asserts that the composer consciously revised his works so that total number of bars was a multiple of 10, 100 and sometimes 1000 resulting in perfect proportions such as, 1:1 or 1:2.

The term Golden Section (GS) refers to the unequal division of a line into two parts such that the ratio of the smaller part to the larger is the same as that of the larger to the original whole. This ratio is approximately 1:1618. It was first documented by the Ancient Greek mathematician Euclid in the *Elements* (c 300 BC) where it was found that this 'division in extreme and mean ratio' appeared frequently in geometry.[150]

150 The *Elements* is a mathematical treatise, one of the most influential works in the history of mathematics, most notably geometry and for over 2000 years has had a huge influence on scientific thinking. The work is attributed to the ancient Greek mathematician Euclid and is a compilation of mathematical research from the previous two centuries including Pythagoras, Archytas and Eudoxus.

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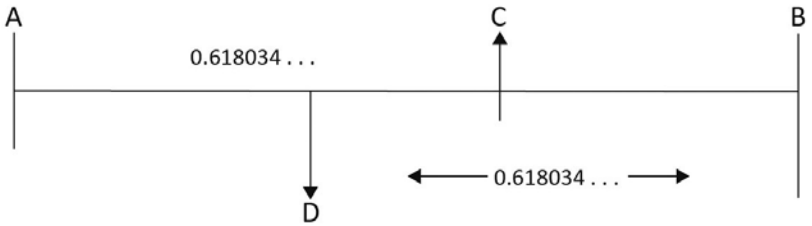
Golden Section



At this time the concept was known simply as the 'section' but during the period of the Renaissance it was taken up by artists, sculptors and architects as a 'divine proportion'; a link was made between the golden ratio and abstract ideas, such as aesthetic beauty and perfection. The educator and composer John Paynter wrote

'The asymmetrical relationship of one-third to two-thirds in measurements of duration or space has immense significance in the history of the world's poetry, literature, visual arts, architecture and music… It is found the world over, in music of every kind, old and new. It is a proportion that is especially satisfying when we observe it in nature - in the shapes and patterning of seashells and fir cones, for example and, perhaps for this reason, it has been consciously emulated in painting and sculpture.… It is discernible in numerous ancient manmade structures and in the `magic' shapes of the pentagon and five-pointed star….'[151]

Mathematically GS is expressed in the Fibonacci series, a summation series in which each number is the sum of the two which precede it.[152] The ratio between the successive terms (0, 1, 1, 2, 3, 5, 8 13, 21, 34, 55 and so on) is an expression of Euclid's golden ratio.

1/1	2/1	3/2	5/3	8/5	13/8	21/13	34/21	55/34
1.000	2.000	1.500	1.333	1.600	1.625	1.615	1.619	1.617

151 John Paynter 'Music in the school curriculum: why bother?' *British Journal of Music Education*. 2002 19:3, 215-226.

152 The Fibonacci series is named after Lionardo Fibonacci the thirteenth- century Italian mathematician who first presented the sequence in his manuscript *Liber abbaci* (1202).

As can be seen, the ratios gradually approach the value known as the golden ratio –

1: 1618033988749894. This is designated by the Greek letter *phi* – Φ, an irrational number.[153] Since the nineteenth century, the number

$$\Phi = 1/2(1 + \sqrt{5}) = 1.6180339887 \ldots.$$

has been called the golden ratio, golden section or golden number. A golden rectangle is a rectangle with sides in this ratio.

The golden ratio can also be applied to durations. As Paynter wrote '… the highly satisfying proportion is also perceptible in defined periods of time, such as musical works, which frequently take off in new and unexpected directions around 0.6 or 0.7 of the overall duration'. It is true to say that the proportions of one third to two thirds are frequently found in music, not least in sonata form (see below). In pop and rock verse and chorus songs, for example, this is the approximate point where the middle eight, a contrasting section, is most frequently to be found.[154] A clear example of GS in pop music can be found in Lady Gaga's 2016 song 'Perfect Illusion' where there is a dramatic key change at exactly this point; the song is 179 seconds long and the key change happens at 111 seconds (179 x 0.618 = 110.622). Paynter argued that GS can also be found in the 12-bar blues but warns that 'It would be unwise to make too much of this, but it is interesting to note how often a composer's intuitive sense of proportion in time structures coincides with the Golden Section divisions'.[155] Many classical pieces feature a dramatic arc somewhere after the midway of the piece, so inevitably this climax will happen close to GS proportions according to the number of bars or the time which has elapsed. Although GS can be viewed and measured in, for example, the façade of a building, Paynter points out that this is more difficult with music; we have to hear the music through. As he wrote, 'Of all mankind's attempts to model perfection, music is, perhaps,

153 An irrational number is a real number which cannot be expressed as an integer or as a quotient of two integers. Irrational numbers have infinite, non-repeating decimals.

154 The middle eight (sometimes referred to as the bridge) is a contrasting section (not necessarily eight bars long) where new material is introduced with, for example, a different arrangement of instruments, and/or different chords.

155 John Paynter. *Sound & Structure* (Cambridge: Cambridge University Press, 1992): 215.

the most subtle. Its meaning is manifest not in objects viewed or touched but in events that can only be experienced in the time it takes to make each one audible'.[156] Consequently it can be difficult to gain any accurate purchase on structural plans in music through listening alone and the use of GS can be audibly imperceptible.

In Mozart's time, sonata form was conceived in three parts: the exposition in which the musical themes are introduced; the development and recapitulation sections where the themes are developed and then revisited. In 1995 John Putz set out to determine whether Mozart had composed the movements of his piano sonatas according to GS. In order to establish this he examined the lengths of two sections: the exposition and the combined length of the development and recapitulation across 16 of Mozart's piano sonatas. Putz found that Mozart was interested in mathematics but that although he 'may have known of the golden section and used it' there is 'considerable deviation from it [which] suggests otherwise'. He came to the conclusion that 'Perhaps the golden section does, indeed, represent the most pleasing proportion, and perhaps Mozart, through his consummate sense of form, gravitated to it as the, perfect balance between extremes.[157]

In some ways this is a chicken and egg situation: Putz points to the argument put forward by the music critic and academic Eduard Hanslick in his 1854 treatise on aesthetics, *The Beautiful in Music (Vom Musikalisch-Schönen)*

> … we should be wrong were we to construe … that man framed his musical system according to calculations purposely made, the system having arisen through the unconscious application of pre-existent conceptions of quantity and proportion, through subtle processes of measuring and counting; but the laws by which the latter are governed were demonstrated only subsequently by science.[158]

Musical analysis by means of GS proportions has aroused much

156 John Paynter, Bernarr Rainbow, *Music Education in Crisis*, Woodbridge, Suffolk: Boydell Press, 2013): 50.

157 John F. Putz. 'The Golden Section and the Piano Sonatas of Mozart'. October 1995 *Mathematics Magazine*

158 Eduard Hanslick, *The Beautiful in Music, translated by Gustav Cohen* (London: Novello, 1891): 151.

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controversy. In her study of GS in the music of Erik Satie (1866-1925), Courtney Adams argued that because the concept itself involves approximation, 'lack of precision provides easy fuel for argument' and consequently 'GS analysis has engendered some scepticism.' As Adams wrote, in compositional analysis it is important to minimize the extent of acceptable deviation from the exact GS ratio because 'GS proportions are perilously close to one-third and two-thirds, fundamental divisions in music.' A useful margin of error is 1 per cent or less. 'Typically, analysts find 2 per cent a reasonable margin, although occasionally a writer will allow a deviation as high as 10 per cent, which I find wholly unacceptable.'[159]

In order to establish the degree of deviation across Mozart's piano sonatas, a similar table was constructed based on Putz's own table but focusing only on first movements (see Table 1).[160] Here the first column identifies the piece by the Köchel cataloguing system, 'a' represents the number of bars in the exposition,[161] and 'b' represents the combined length of the development and recapitulation (without including any codas). The column 'c' shows the number of bars that would have been used if the movement had adhered strictly to the proportions of GS and the final column shows the percentage difference from the actual number of bars in the Mozart sonata.

159 Courtney S. Adams. 'Erik Satie and Golden Section Analysis'. *Music & Letters* , May, 1996, Vol. 77, No. 2: 242-252, Oxford University Press: 243-244.

160 The first movement of K. 331 in A has been omitted because it is not in sonata form, rather it is a set of variations.

161 Although the expositions of Mozart's piano sonatas are normally repeated in performance, Putz gives the number of bars of the exposition without repeats.

Table 1 Golden Section proportions in the first movement of Mozart's piano sonatas

Köchel number	a - No of bars in exposition	b - No of bars in development + recapitulation	a + b	c - b according to Golden Section	% difference
K. 279 in C	38	62	100	61.8	< 1%
K. 280 in F	56	88	144	90.0	> 2%
K. 281 in Bb	40	69	109	67.4	> 2%
K. 282 in Eb	15	18	33	20.4	< 2%
K. 283 in G	53	67	120	74.2	> 1%
K. 284 in D	51	76	127	78.5	> 3%
K. 309 in C	58	97	155	95.8	> 1%
K. 310 in A minor	49	84	133	82.2	> 2%
K. 311 in D	39	73	112	69.2	> 7%
K. 330 in C	58	92	150	92.7	< 1%
K. 332 in F	93	136	229	141.5	> 3%
K. 333 in Bb	63	102	165	102.0	Exact
K. 457 in C minor	74	93	167	103.2	> 9%
K. 533 in F	102	137	239	147.7	> 7%
K. 545 in C	28	45	73	45.1	< 1%
K. 570 in Bb	70	130	200	123.6	> 5%
K. 576 in D	58	160	218	134.7	> 5%

In terms of adherence to GS proportions, five of the piano sonata movements stand out – K. 333 is exact according to GS proportions, and K. 279, K. 283, K. 309, K. 330 and K. 545 all have less than 1% difference. As was mentioned earlier, Adams considered a difference smaller than 2% to be a reasonable margin. This would mean that K. 282 would also fit into this category giving a total of 6/17 movements - (35%). On the other hand some movements, K. 311, K. 570, and K. 576, for example, show what Putz describes as 'considerable deviation' leading us to the same conclusion as Putz, that is that Mozart lent towards the 'pleasing proportion' of GS and gravitated towards it intuitively.

Before we move onto the survey of in the music of Erik Satie, it is revealing to examine a piece of music which follows GS proportions exactly and was written only 34 years after Mozart completed his final piano sonata. In her analysis of Schubert's lied 'Du bist die ruh' (You are my calm) (1823), the music theorist Pozzi Escot discovered that, discounting the first seven bars of solo piano introduction, the song falls into two distinct parts. It has a total of 75 bars (82 - 7), Part I (bars 8- 53) is 46 bars long and Part II (bars 54-82) is 29 bars long. This means that Part II starts at exactly at the point of Golden Section.[162] Furthermore, at this point the music is marked by a dramatic increase in tension.

75 x 0.618 = 46.35

'Du bist die ruh' is a setting of a love poem by Friedrich Rückert. The first part tells of calm repose, the opening bars (below) have a feeling of stillness reflected in the simple harmony and quiet dynamic. The vocal line opens with the following four bars repeated.

When the point of GS is reached there is a dramatic change as the mood is transformed from one of calm repose to deep yearning. There is a tremendous tonal shift as the harmony modulates from Eb to Cb and we hear the first crescendo as the vocal line rises to the highest note in the song. This is followed by a dramatic silence.

162 Pozzi Escot. *The Poetics of Simple Mathematics in Music*. (Cambridge: Publication Contact International, 1999).

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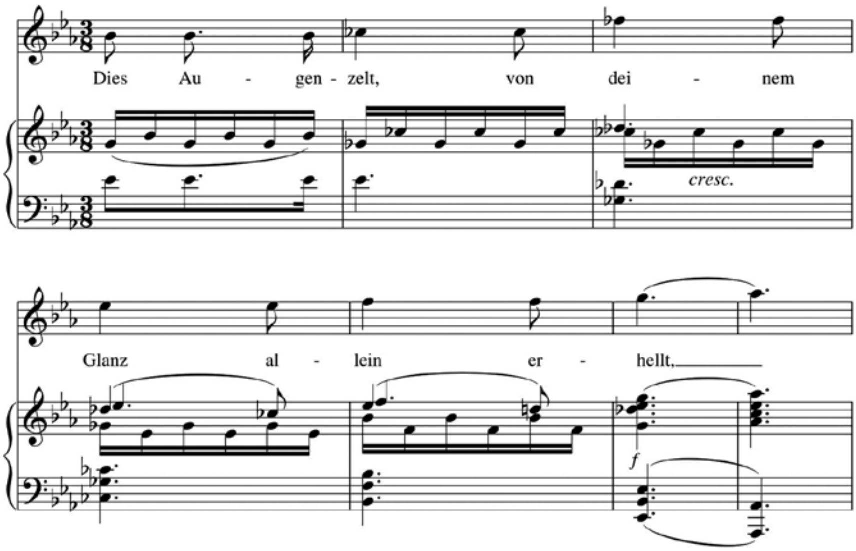

Erik Satie and Golden Section

Returning to the GS analyses of Adams, in her survey of this mathematical device in the music of Erik Satie, Adams found four multi-movement sets and six individual movements using GS proportions 'that are exact or within a deviation of one per cent.[163] The works belong to both the early (1887-92) and late (1914-19) periods of Satie's output. She set out to find an explanation as to these two clusters, aiming to establish whether the appearance of GS was the result of 'conscious use, coincidence or, as some would argue, instinctive application.' In summary, she found that although it was possible that Satie had learnt about the principle from Debussy in late 1891 and had experimented with it, there was no convincing answer with regard to the early pieces, and that the use of GS in these was largely either instinctive or coincidental. This was reinforced by the 'absence of any calculations in the sketchbooks over this long period' However, she found that in the later period 'there is ample evidence of his close association with a number of people who were demonstrably aware of GS' and that the GS proportion appears regularly in Satie's music between 1914 and 1919, including the

163 The sets are the *Trois Gymnopedies, Les Trois Valses, Avant-dernieres pensees* and the first three *Nocturnes*. The individual movements are *Le Fils I, Sonneries I* and *III* and *Trois melodies*.

'overall movement proportions of most of his three-movement piano works' but also the internal form of each of the three songs in his song cycle *Trois Mélodies* of 1916. She found that the usage here was 'both too consistent and too precise to attribute to chance or instinct.'[164] The three songs of *Trois Mélodies* are 'La Statue de bronze', 'Daphénéo' and 'Le Chapelier'. 'Daphénéo' is used here as an example (see below). The song opens with both the vocal line and the piano accompaniment formed largely of quavers.

'Daphénéo' is 39 bars long so we would expect the GS division to occur shortly after bar 24 and for there to be a new turn in the music. As Adams points out, 'the long GS (after bar 24) marks the end of two bars which include the lowest note in the piece; moreover it introduces a sudden semiquaver motion that leads to a new register at the reprise.' So a case could be made here for the use of GS.[165]

164 Adams. 'Erik Satie and Golden Section', 250-251.

165 Adams, 'Erik Satie and Golden Section', 248.

Erik Satie *'Daphénéo'* bars 17 - 28

Béla Bartók, Claude Debussy and Golden Section

Other claims for the use of GS in music, notably by Bartók and Debussy, have aroused more controversy. In his book *Béla Bartók: An Analysis of His Music* (1971), the Hungarian music analyst Ernő Lendvai argued that the composer used formal principles based on GS and the Fibonacci series, in, for example, his *Sonata for Two Pianos and Percussion* and *Music for Strings, Percussion and Celesta* which were both composed in 1937.[166] Lendvai studied the latter piece in detail and cited, amongst other examples, the use of Fibonacci numbers as structural devices in the first movement. Unfortunately he made a few errors and fudged some of his calculations. The music analyst Roy Howat discredited the work and he countered these claims in his own detailed analysis of the work which exposed some of the inaccuracies. He found for example that 'numbers of bars begun are sometimes confused with bars completed'. Howat argued that the use of proportional systems was most likely to have been subconscious given that there was 'no definite

166 Ernő Lendvai. *Béla Bartók: An Analysis of His Music*. (London: Kahn and Averill, 1971).

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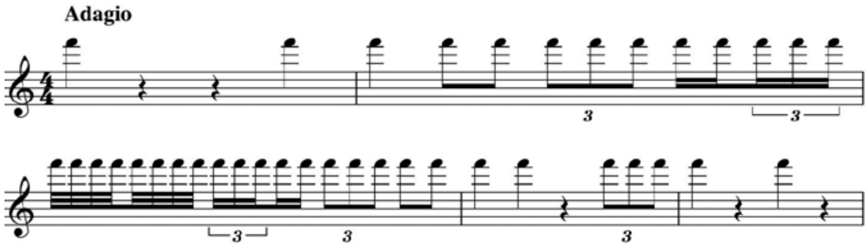
proof of conscious application'.[167] He did however make a case for the use of Fibonacci numbers in the slow third movement of Bartók's *Music for Strings, Percussion and Celesta* (music to familiar to those who have seen the film *The Shining* where it is used to create an eerie atmosphere). The Fibonacci series is quite clear to see in the xylophone solo of the opening bars where the rhythmic pattern uses 1–2–3–5–8–5–3–2–1.



Howat found further examples of the GS and Fibonacci series in this movement by dividing the movement up into crotchet beats, rather than bars as Lendvai had done, revealing clear structural examples.[168]

The question remains as to whether Bartók was making conscious use of these mathematical proportions. The musicologist Paul Griffiths is very dismissive writing that the 'appearance of the Fibonacci series in a rhythmic pattern at the start of the slow Movement of Bartók's *Music for Strings, Percussion and Celesta* (1–2–3–5–8–5–3–2–1) is suggestive, but no more'.[169] Lendvai acknowledged that Bartók said very little about his compositional techniques stating 'Let my music speak for itself; I lay no claim to any explanation of my works'.[170] As Laszlo Somfai wrote, in his research into Bartók's extant sketches and autograph, he did not find 'a single calculation of the proportion of a composition - with Fibonacci or other numbers'.[171] Howat wrote that Bartók was secretive and reluctant to divulge the 'use of highly

167 Roy Howat. 'Bartók, Lendvai and the Principles of Proportional Analysis' *Music Analysis.* Vol. 2, No. 1 (March 1983): 69-95.

168 Howat, 'Bartok and Proportional Analysis': 80-82.

169 Paul Griffiths, 'Numbers in music' in *Grove Music.*

170 Mario Livio. *The Golden Ratio: The Story of Phi, the World's Most Astonishing Number.* (New York: Broadway Books, 2002): 190.

171 Laszlo Somfai. *Béla Bartok: Composition, Concepts and Autograph Scores*. (Berkeley: University of California Press, 1996).

abstract constructions in what many critics already regarded as over-cerebral music' but he goes on to point out that 'More positive manuscript evidence of proportional calculations exist in Bartók's transcriptions of folk music' citing as evidence Manuscript 80FSS1 in the New York Bartók Archive, a sketchbook devoted largely to Bartók's first drafts of Turkish folk songs. These include a metrical sequence of numbers which is easily identifiable as the Lucas summation series (1, 3, 4, 7, 11, 18 …) where, in the same way as the Fibonacci sequence, each term is the sum of the two previous terms, and the ratios of successive terms approach the Golden ratio.[172] Howat argued that this demonstrates that the song's 'metrical organization is analysed wholly in terms sequence numbers'.[173]

In 1983, Howat went on to make a detailed case for GS as a structural device in Debussy's music in his book *Debussy in Proportion: A Musical Analysis* where he set out to demonstrate Debussy's use of 'intricate proportional systems', in particular the Golden Section .[174] His study included detailed analyses of the piano pieces *Reflets dans l'eau* (1905), *L'isle joyeuse* (1904), and the symphonic poem *La mer* (1908), and demonstrated how the pieces are built around GS and the Fibonacci series. In *Reflets dans l'eau*, the first of three pieces in Debussy's first volume of *Images*, as part of the detailed analysis Howat found that the principal climax at bars 56-61 tallied with the overall point of GS i.e. after 58 bars out of a total of 94. He also discovered that Fibonacci numbers are often found at 'strategic places' such as in the 55 bars of introduction to the final section, 'Dialogue du vent et de la mer', of *La Mer* (where the bar groups follow the Fibonacci numbers 3, 5, 8, 13, 34 and 55) and the 34 bars of 'build-up to the climactic coda' of *L'isle joyeuse* (bars 186-219). Howat considered the question of whether Debussy's proportional patterns were 'designed consciously' or 'intuited unconsciously' but came to no definite conclusion he did however acknowledge that Debussy very probably knew about GS and Fibonacci through a series of articles in which the mathematician Charles Henry discussed such structural principles in

172 Lucas numbers are named after the French mathematician François Édouard Lucas (1842-91) who launched the *American Journal of Mathematics* in 1878 with a long paper dealing with the Fibonacci sequence along with the sequence starting 1, 3 which he named after himself.

173 Howat, 'Bartok and Proportional Analysis,' 85.

174 Roy Howat. *Debussy in Proportion: A Musical Analysis* (Cambridge: Cambridge University Press, 1983).

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the Symbolist journals which Debussy 'read avidly'.[175] Howat later added a caveat 'How consciously this came about is unproven, and the subject can still be contentious; its main interest lies in linking different aspects of the structure into a naturally balanced dramatic flow'.[176]

In the 1940s, the Russian/American teacher and composer Joseph Schillinger (1895-1943) devised his own system of composition based on mathematical principles.[177] His thinking behind this was that a mathematical approach would be more natural because nature was intrinsically mathematical. In his words, 'Music imitates nature, and particularly the forms of motion in the universe, i.e. the growth and evolution of natural forms. The forms of organic growth appeal to us as a form of beauty when expressed through an art medium'.[178] In his books *The Schillinger System of Musical Composition* (1941) and *The Mathematical Basis of the Arts* (1948) he suggested creating pitch structures based on the Fibonacci sequence.[179] His idea is based on counting semitones where each semitone represents one unit. So, for example, starting on C and moving up one semitone would mean that the second note would be Db. Schillinger's system alternated between ascending and descending intervals. In this way the Fibonacci series 1, 1, 2, 3, 5… starting on C would create the following sequence of notes.

C up one semitone **Db**, down two semitones **B**, up three semitones **D**, down five semitones **A,** as below.

175 Roy Howat. *The Art of French Piano Music: Debussy, Ravel, Fauré, Chabrier*. (London: Yale University Press, 2009).

176 François Lesure and Roy Howat, 'Debussy' in *Grove Music*.

177 After Joseph Schillinger had emigrated to the USA he taught his mathematical methods to a large number of pupils, many of these were popular musicians and included George Gershwin, Glen Miller and Quincy Jones amongst their number. Another pupil, Lawrence Berk, became the founder of the Berklee College of Music where Schillinger's methods were taught in its early days. Schillinger's ideas included various mathematical ways in which to produce melodies, rhythms and chords and, although they were littered with miscalculations and misunderstandings of mathematical concepts, for many years they were in high demand.

178 Joseph Schillinger: *The Schillinger System of Musical Composition* (1941).

179 Joseph Schillinger: *The Schillinger System of Musical Composition* (1941) and T*he Mathematical Basis of the Arts* (1948).

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Darmstadt International Summer Courses for New Music in the 1950s and 1960s

In 1854, Eduard Hanslick had argued in *The Beautiful in Music* that 'No mathematical calculation ever enters into a composition, be it the best or the worst. Creations of inventive genius are not arithmetical sums'[180] One hundred years later, however, there were several composers associated with the centre of new music in Darmstadt who made conscious and open use of GS and the Fibonacci series. Central to the Darmstadt International Summer Courses for New Music were premieres of new works and the teaching of composition.[181] It soon became what Alex Ross describes as the 'principal showcase for the avant garde'.[182] In his 1957 lecture on the development of serial technique the Italian composer Luigi Nono (1924-1990) coined the term 'Darmstadt School' for a group of composers that included Bruno Maderna, Karlheinz Stockhausen, Pierre Boulez and Nono himself.

Stockhausen (1928-2007) went on to make extensive use of the Fibonacci series in his music of the 1960s and, according to the American music theorist Jonathan Kramer, used it 'prominently and systematically'. Kramer argues that in Stockhausen's music the Fibonacci series contributes 'more significantly to the total form than it does in the [music of] Bartók'; the Fibonacci series is used proportionally in many ways, some simple and some complicated'. In *Klavierstuck IX* (1961), for example, majority of bars have Fibonacci-related time signatures and in the coda, each bar contains either 1, 2, 3, 5, 8, 13, or 21 attack points. A larger scale use of Fibonacci proportions can also be found in *Adieu* (1966) for wind quintet where the durations of all the bars 'except the five which contain tonal references and the four with unmeasured silences' are given by Fibonacci numbers from 1 to 144. Kramer wrote that the 'properties of the series are readily heard because each

180 Hanslick, *The Beautiful in Music*, 92.

181 Darmstadt International Summer Courses for New Music (*Ferienkurse fiir neue Musik*) were initiated in 1946 by Wolfgang Steinecke.

182 Alex Ross. *The Rest is Noise*. (London: Fourth Estate, 2008): 391.

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duration contains a largely static … unfolding of a sonority' which changes when the music moves to the next bar.

In Nono's cantata *Il Canto Sospeso* (1956) he utilised the Fibonacci series in a different way. Rather than generating formal proportions Nono determined individual note lengths by means of the series. In the second movement, for example, the durations of notes are generated by the Fibonacci numbers 1 2 3 5 8 13.[183] The Danish composer Per Nørgård also experimented with the properties of the Fibonacci series in some of his music. In the programme note for his orchestral piece *Iris* (1968) he compares his use of the Fibonacci series to the way it can be found in nature:

> The basic idea is … a network of lines where each one represents a rather simple melody or rhythm. The whole thing is not unlike the flowers of the daisy, where you see 21 spiralling lines moving in one direction and 34 spiralling lines in the other, in other words the proportion of The Golden Mean (as expressed in the Fibonacci series.[184]

The Austrian composer and writer Ernst Krenek (1900-1991), was a regular attendee at Darmstadt. In his 1964 piece *Fibonacci Mobile* Op. 187 for string quartet and piano duet, he uses the Fibonacci series in the construction of the proportions.

Although the ideas of the radical Greek composer Iannis Xenakis (1925 – 2001) were often at odds with the serialism of the Darmstadt School, his orchestral piece *Metastaseis* (1954) was the subject of a lecture given there in 1962 by György Ligeti (1923-2006). Following his arrival in Paris in 1947, Xenakis had worked as an architect and engineer for 12 years with Le Corbusier where proportion was central to his work (see also pages 187-8). This was to have a strong influence on the composition of *Metastaseis*. In the words of Xenakis, "In my composition *Metastasis* … the role of architecture is direct and fundamental by virtue of the Modular. The Modular was applied in the very essence of the musical development"[185] Here he is referring to a system devised by Le Corbusier, an architectural approach

183 Jonathan Kramer. 'The Fibonacci Series in Twentieth-Century Music'. *Journal of Music Theory*, Spring, 1973, Vol. 17, No. 1 : 110- 148.

184 Per Nørgård, programme note for his orchestral piece *Iris*, 1988.

185 Sharon Kanach. *Iannis Xenakis. Music and Architecture*. (New York: Pendragon Press, 2008).

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to form and proportion and used by Xenakis to shape pitch envelopes and time structures in *Metastaseis*. *Metastaseis* is built in four sections. The first section begins on a sustained single note G played very quietly by all 46 strings gradually sliding outwards by means of slow glissandi as the dynamic intensity increases, arriving at a massive note cluster for full orchestra with each instrument sustaining its own note. This large cluster is treated as one sonic unit where note lengths are derived from the Fibonacci/Modulor series by means of dynamic and timbral changes.[186]

*P*roportional parallelism in the music of J S Bach

As we have seen, GS is closely linked to proportion therefore readers may also be interested in the theory of proportional parallelism in the music of J S Bach that has been put forward by the musicologist Ruth Tatlow. This asserts that 'all Bach's revised fair copies and published collections have three characteristics in common: the total number of bars are a multiple of ten, one hundred, and sometimes one thousand; there are perfect proportions such as, 1:1 or 1:2 on at least three different levels; and there is an embedded signature.' Tatlow discovered that the 'total number of bars in the six Brandenburg Concertos is exactly 2220, Concertos 4, and 5 having an exact total of 1110 bars, and Concertos number 1, 2, 3, and 6 also with 1110 bars, creating a parallel 1: 1 (1110: 1110 bars) in 1: 2 (4: 2 concertos)'. Bach created these parallel proportions when he revised his works and collections in preparation for publication by adding a 'few bars to movements and movements to collections'.[187]According to Tatlow, proportional parallelism offers an explanation for these revisions, hitherto unexplained and with no apparent musical objective; she argues that Bach was motivated to make these changes because of his Lutheran beliefs in the theory and practice of music that leant heavily on universal harmony.[188].

186 James Harley, *Xenakis. His Life in Music*. (London: Routledge, 2004): 10.

187 Daniel R Melamed.'Parallel proportions in J S Bach's music'. Published online by Cambridge University Press: 05 February 2021

188 Ruth Tatlow. *Bach's Numbers: Compositional Proportion and Significance*. (Cambridge: Cambridge University Press, 2015).

Tatlow's theory challenges a long-held and influential theory of the German Bach scholar and theologian Friedrich Smend (1893-1980) which argues that J. S. Bach's works contain hidden symbolic references based on the German 'natural-order' number alphabet (A= 1 … Z=24).[189] Smend's work spawned many further studies of Bach's presumed extensive use of number symbolism which was seen as in keeping with his 'mystical' musical outlook. The response to Tatlow's work has been mixed, with some scholars being critical, notably Daniel Melamed.[190] Alan Shepherd explored Tatlow's theory further in *Let's Calculate Bach* (2021) which sets out to apply theories of probability and statistical analysis to numbers in music.[191]

Golden Section and the associated Fibonacci series give rise to satisfying natural proportions. Many examples of their use can be found in music, sometimes these adhere strictly to the exact ratios and at other times they are approximations. The question remains as to whether they are used by composers consciously or subconsciously. A study of first movement sonata form across Mozart's piano sonatas reveals that, although more than a third of them fall within what might be considered to be a reasonable GS margin, it is more likely that Mozart lent towards the pleasing proportions intuitively through his innate sense of form. Cases have been made for various pieces using GS by Satie, Bartók and Debussy, but these have been inconclusive in the absence of documentary evidence to prove that they were based on mathematical calculations. This is not the case with several composers associated with the centre of new music in Darmstadt during the 1950s and 1960s; Stockhausen, Nono, Krenek and Nørgård, for example, all both consciously and openly integrated the use of the Fibonacci series into their compositional techniques.

189 Friedrich Smend. *Luther und Bach*. (Germany: Verlag Haus und Schule, 1947).

190 Daniel R. Melamed, "'Parallel Proportions' in J. S. Bach's Music," Eighteenth-Century Music 18, no. 1 (2021): 99–121. 4 See Melamed, "'Parallel Proportions,'" 100–103.

191 Alan Shepherd. *Let's Calculate Bach: Applying Information Theory and Statistics to Numbers in Music* (Cham, Switzerland: Springer, 2021).

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