11 Randomness and chance

Random numbers in mathematics

GENERALLY SPEAKING, RANDOMNESS means having no specific pattern, purpose or principle of organization. Random sequences of events have no order and single random events are unpredictable. Chance is the unknown and unpredictable element that causes an event to have one outcome rather than another. Nowadays random numbers are computer generated. They are widely used in both sample selection and in allocating treatments to units in designed experiments. Randomized control trials are used on two groups – a study group and a control group. In medical trials, for example, scientists test the study group (those who receive a new treatment) and a control group (those who are given an existing treatment or a placebo). Trials could be biased if it is known which group the patient belongs to so random selection is used to avoid this bias.

Just as random selection is used to avoid bias in scientific experiments by taking control away from the project leader, so too in music the use of chance operations means that total control is taken away from the composer. The terms indeterminacy, chance music and aleatoric music are often used interchangeably to describe this random element. Music using chance processes falls into three main groups: the use of random procedures to produce a fixed score; indeterminate notation, including graphic notation and texts to be interpreted by the performer(s); and scores where the indeterminate element arises from choices made by the players about elements of the performance.

This chapter explores the different ways in which composers have used randomised elements in their music from the use of rolls of dice by eighteenth-

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century composers to the computerised chance operation of Iannis Xenakis who coined the term 'stochastic music'. The leading composer of aleatoric music was undoubtedly John Cage who used chance procedures in almost all the music he created after 1951.

The use of dice in eighteenth-century novelty music

Dice (singular: die) were one of the earliest means of generating random numbers and examples can be traced back to 24,000 BC. During the eighteenth century a fashion developed where dice were used to introduce an element of chance into musical compositions. This genre was known as *Musikalisches Würfelspiel* (musical dice game). It was a system for randomly generating compositions from pre-composed bars of music. The games consisted of a number of pre-composed bars along with a method for selecting their precise sequence usually the throwing of a number of dice. Random numbers were used to order the melodic fragments thus generating a new piece. One of the first such games was published by the German composer and music theorist Johann Kirnberger (1721-1783) in 1757. *Der allezeit fertige Polonoisen und Menuettencomponist* (The at all times prepared composer of polonaises and minuets.) comprises a 'score' and a sequence of possible bars for both of the musical forms. The sequence of bars is determined by the throwing of either one or two dice.

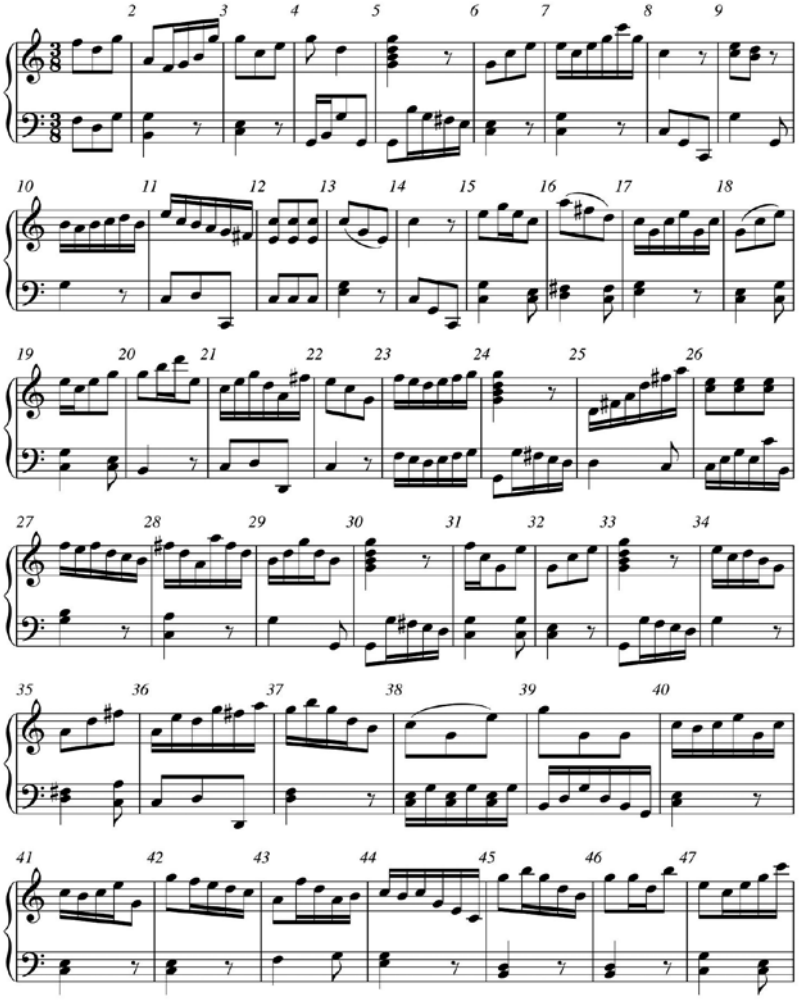
Some pieces were attributed to well-known eighteenth-century composers such as C P E Bach, Mozart and Haydn. One such piece attributed to Mozart, but not authenticated, is *Anleitung zum Componieren von Walzern so viele man will vermittelst zweier Würfel, ohne etwas von der Musik oder Composition zu verstehen* (Instructions for the composition of as many waltzes as one desires with two dice, without understanding anything about music or composition) (1792). Rolls of the dice are used to select bars of music to be used in sequence and then played as a complete waltz. Each number rolled corresponded with a bar number given in a table. There are 176 numbers given in two tables (one table for the first eight bars of the waltz, and the second table for bars 9 to 16 of the composition). Each of the numbers from 1 to 176 corresponds with a bar of music. To select the first bar of the waltz, two dice are thrown and then the corresponding bar is chosen from the first

column, for the second bar two more dice are thrown and the appropriate bar is selected from the second column and so on. Given that there are 11 different options for each of the 14 bars along with two options for one of the bars (bar 8) and only one option for the final bar (bar 16).[192] This means that there are 2 x 11^{14} possible variations on the waltz – 759, 499, 667, 166, 482 trillion. Here is the first table and the first 47 bars of the music.

Mozart dice game

	A	B	C	D	E	F	G	H
2	96	22	141	41	105	122	11	30
3	32	6	128	63	146	46	134	81
4	69	95	158	13	153	55	110	24
5	40	17	113	85	161	2	159	100
6	148	74	163	45	80	97	36	107
7	104	157	27	167	154	68	118	91
8	152	60	171	53	99	133	21	127
9	119	84	114	50	140	86	169	94
10	98	142	42	156	75	129	62	123
11	3	87	165	61	135	47	147	33
12	54	130	10	103	28	37	106	5

192 Giovanni Albini. 'Combinatorics, probability and choice in music composition'. Bridges 2018 Conference:



The appeal of the games was that they appeared to make it possible for anyone to be able to compose a piece of music. In fact composers used their knowledge of compositional procedures, harmony and formal design, coupled with a basic understanding of mathematics to design the games in such a way that whatever options were arrived at 'by chance', the resultant composition would sound convincing. Many bars are identical (bars 8 and 13, for example), many have the same notes in the left hand (3, 6, 32, 40, 41 and 42, for example) and the number of chords implied in the harmony is limited. So, for example, bars 3, 6, 7 and 8 are all based on the tonic chord of C major (C

E G), whereas bars 2, 4 and 5 are all based on the dominant chord - bar 2 is based on G (G B D) and bars 4 and 5 have an added 7th (G B D F#). This means that the table of bar numbers can be manipulated so that the devised piece makes harmonic sense. An example of this is Column H; this gives the selection for the eighth bar of the piece. According to the number of the dice thrown the bar selected will be either 30, 81, 24, 100 or 107, and so on. In fact each of these bars is based on a dominant chord with either an F natural or an F#. This implies an imperfect cadence which, in this Classical style of music would be expected half way through the 16 bar melody at the end of an eight bar phrase. Similarly, all the possible options for bar 16 (14, 73, 89, 170 and so on) are identical, meaning that whatever dice are thrown, the piece ends on a tonic chord as would be expected and sounds finished.

These compositions involving chance were novelties. Until the twentieth century, Western classical music had traditionally been a field in which chance had little or no role to play. As John Paynter writes, 'Musical action involves feeling, responding, thinking and making' and until the early years of the twentieth century, mainstream composers took this to mean that they should 'have overall control of their musical ideas in order to express the consciousness of their being'.[193] When chance music, often known as aleatoric music, emerged in the mid-twentieth century it challenged the traditional idea of a composition being a closed entity fixed by its composer; elements of chance which were undetermined by the composer were introduced into the process of composition and/or in performance. As such it explored what Paynter describes as 'ways of creating music which, if possible, would be divorced from the will and the taste of the composer – the very opposite of the traditional expressing and communicating objective'. Composing rather than 'being a matter of putting together all the details of a carefully calculated and evaluated structure' instead 'set up opportunities for "sound events" to happen'.[194]

> All sounds and any sounds could come in and find a place in the music; and the accidental structures they created could be as delightful in their own way as the random sounds of nature – which we can enjoy without their having been 'Composed'. [195]

193 John Paynter. *Sound and Structure.* (Cambridge: Cambridge University Press, 1992): 166.
194 Ibid
195 Ibid

Chance in twentieth-century music – Charles Ives, Henry Cowell, Morton Feldman and John Cage

The American composer Charles Ives (1874–1954) allowed a greater degree of freedom for performers in some of his pieces through the experimental use of alternatives, free passages and obscure notations inviting the performer's interpretation. A friend of Ives, the American composer Henry Cowell (1897-1965) continued this line of experimentation in aleatoric music. His *Mosaic Quartet* (*String Quartet No. 3*, 1934) is an experiment in musical form; a collection of five short movements in five different tempos where the order of the movements, and the number of times each is played, is left up to the performers. The piece also calls for some improvisation, another chance element. However it was Cowell's sometime pupil, John Cage (1912-1992), who, starting in the early 1950s, was to make the fullest exploration of chance operations. Cage's exploration of chance processes was partly inspired by *Extensions 1* (1951) by Morton Feldman (1926-1987) where sounds are written as numbers on graph paper, the figures indicating only the number of notes to be played and the register in which they are to be performed. Cage admired Feldman's use of 'whatever sound comes along'[196] and in his 'Lecture on Something' (written in early 1951), he described Feldman's graph pieces as having 'changed the responsibility of the composer from making to accepting.'[197]

In 1951 Cage completed his *Music of Changes*, a piano piece whose title makes reference to the *I Ching*, the Chinese oracle text sometimes known as *The Book of Changes*. The *I Ching* is concerned with divination – predicting the future through a systematic process, in this case using random processes to produce and interpret a series of symbols. The *I Ching* represents the binary poles of reality as Yin and Yang (negative/positive, night/day, winter/summer…) these are often discussed as female and male in the text of the *I Ching*. In the *I Ching* images are selected at random from a set of 64 by means of tossing yarrow sticks or coins. The *I Ching* is structured around an idea which is essentially combinatorial, that is it uses principles which stem from

196 Barry Russell and Julia Winterson. *Everything we do is Music. Cross curricular Experiments Based on the Music of John Cage*. (London: Edition Peters, 2016).

197 John Cage. 'Lecture on Something' in *Silence. Lectures and Writings by John Cage*. (Middletown, Conn.: Wesleyan University Press, 1961): 129.

combinatorics, the mathematical study of the enumeration, combination and permutation of sets of elements and the mathematical characteristics of their properties. A broken line represents Yin, and an unbroken line represents Yang. These lines are combined in sets of three to produce the eight basic trigrams, which are in turn combined to give 64 hexagrams. Each hexagram describes a different state of mind and is accompanied by an explanatory text. The text discusses each of the six lines as separate entities as well as explaining the meaning of the hexagram as a whole. Once the hexagram is cast, the *I Ching* is consulted for insight into its meaning. Cage made extensive use of the *I Ching* in the composition of *Music of Changes* using chance operations to create charts for tempi, dynamics, sounds and silences, and durations.

He went on to use chance operations in many other compositions and designed several different chance-controlled systems. In the *Music for Piano* series (1952–6) he translated the imperfections on paper into musical notes by the application of staff lines and clefs. *Atlas Eclipticalis* (1961-2) used templates drawn from maps of the constellations. *Concert for Piano and Orchestra* has no score, but rather consists of 63 highly detailed parts. It was composed using chance operations as well as the use of the imperfections found in the paper upon which the music was written. The notation of each part uses a system wherein space is relative to time. The amount of time is determined by the musician and then altered during performance, by the conductor, whose arms simulate the movement of the hands of a clock. The pianist may play the material in whole or in part, choosing any notations, elements, or parts, and playing them in any order.

In *HPSCHD* (1969) Cage used a computer to generate random musical material for a multi-media event that involved harpsichords and computer-generated sounds. The first performance lasted five hours and included slide projections. Three large posters were created featuring images chosen by chance operations, copies were sold at different prices arrived at by using the *I Ching*. The music comprises 20-minute solos for 1-7 amplified harpsichords and tapes for 1-51 monaural machines distributed unpredictably to make an indeterminate concert of any length having 2 - 58 separate channels with loudspeakers around the audience. The harpsichord solos used music by

composers including Mozart (primarily the music of the aforementioned Mozart dice game), Beethoven, Chopin and Schoenberg.

In 1952 John Cage was one of a group of artists, musicians, poets, film makers and dancers to take part in what is said to be the first 'happening', a title which covers a range of multi-media performance activities where many things happen simultaneously with no narrative structure. Cage's first happening was at Black Mountain College. For that event, later called *Theatre Piece No. 1*, Cage gave all the participants time-brackets during which they were to dance, read poetry, show film and/or play music. The time brackets were used to specify the intervals during which sounds must start and stop. He then went on to use the concepts of both time brackets and happenings in many other works.

Cage was a prolific composer and most of his mature pieces included some element of chance. He stated that his goal was to be 'free of individual taste and memory'.[198] In some ways this is similar to the use of random selection to free bias in scientific experiments by taking control away from the project leader. The introduction of chance into a piece of music undermines the idea that creation requires a definite choice at every level on the part of the composer. With Cage this abandonment of control reached its extreme point *4′ 33″* (1952),or 'My silent piece,' as Cage called it, an empty structure in which the only sounds – those of the environment – are the most full of possibilities. As Cage once stated in his lecture 'Indeterminacy'

…the purpose of this purposelessness music would be achieved if people learned to listen; that when they listened they might discover that they preferred the sounds of everyday life to the ones they would presently here in the musical program; that was alright as far as I was concerned.[199]

Fluxus, Terry Riley and Cornelius Cardew

Cage's use of chance operations, particularly those used in multi-media performances, influenced many other composers. As Michael Nyman writes 'By the late sixties indeterminacy … had made itself available to a larger

198 Cage. *Silence*, 59.

199 Cage. *Silence*, 267.

number of people with a wider range of abilities and experience'.[200] It was central to the work of the New York group Fluxus which formed in 1963 and went on to stage performances which were 'characterised by simplicity, experimentalism, chance, playfulness and humour'.[201] The indeterminate scores of Cage and his contemporary Christian Wolff often demanded technical and musical expertise, but with Fluxus this was no longer the emphasis, instead it encouraged unskilled players thereby, it could be argued, increasing the element of randomness. An advertisement for a concert in 1965 read as follows

> FLUXORCHESTRA PERFORMS 20 WORLD PREMIERS! Of avant-gagist music, ying yang music, Donald Duck music, anti-neobaroque music, pataphysical music, no music, La Monte Young conducting an orchestra of twenty unskilled instrumentalists.[202]

Fluxus counted amongst its members Dick Higgins, La Monte Young, Joseph Beuys, Yoko Ono and Terry Riley and eventually became an international movement with offshoots in Germany and elsewhere. In 1964 the American composer and performer Terry Riley (b 1935) went on to compose one of the most frequently played pieces where chance is central to its performance – *In* C.

The score of *In* C comprises 53 'repeating patterns' which should be played in consecutive order starting with 1 and ending with 53. All performers (any number playing any instrument) play from the same part apart from the 'pulse'. A steady quaver pulse is played on the top octave of a piano, marimba or vibraphone. 'Each performer decides for himself when to enter' and 'is free to move from figure to figure at his own rate'.[203] This element of choice makes the detail of each performance of *In* C unique. It could also be regarded as a precursor of the Minimalism movement.

In London Cornelius Cardew (1936-1981) formed the Scratch Orchestra which had a similar philosophy to that of Fluxus. All members were encouraged to participate on an equal footing regardless of skill or previous

200 Michael Nyman. *Experimental Music. Cage and Beyond*. (Cambridge: Cambridge University Press, 1999): 110

201 *Every Day is a Good Day. The Visual Art of John Cage*: 62.

202 *Village Voice*, 23 September 1965.

203 Score of Terry Riley *In C*.

experience. Formed in 1969, it brought together trained and untrained musicians from various backgrounds and was dedicated to free musical exploration. The band's repertoire included simple classics as well as Cardew's own compositions. Probably the most famous of his compositions is *Treatise* (1967) which has a totally graphic score (apart from the very occasional musical symbol). Cardew also worked as a graphic designer and the score – a melange of geometrical forms – raised graphic notation to the level of visual art as does the score for *Octet '61 for Jasper Johns* (1961). At the same time the score of *Treatise* provided the performer with little or no information as to how it was to be interpreted, consequently the ways that the sounds can be realized are endlessly diverse. As Cardew writes in the score 'Any rigidity of interpretation is automatically thwarted by the confluence of different personalities'. There are many recorded interpretations of *Treatise* and these are not confined to performances by experimental music groups. In 1999 the American rock band Sonic Youth included page 183 on their album *SYR4: Goodbye 20th Century*.

A key piece in the repertoire of the Scratch Orchestra was Cardew's *The Great Learning* (1971) based on the first chapter of the Confucian text with the same name. It was developed collectively by an experimental music group which Cardew ran at Morley College in South London. The piece consists of seven extensive movements - 'Paragraphs'- and is a blend of improvisation, formal composition and Maoist propaganda. In order to fulfil his intention that anyone could enjoy collective music-making, Cardew uses a variety of notations predominantly graphic notation and simple verbal descriptions of performance processes. 'Paragraph 7' is for voices alone, any number of trained or untrained singers. The text instructs the performers to start on a random note and with each breath to move to a new note sung by one of their neighbours.[204] This means that in performance, having started with a completely unpredictable chordal texture, the pitch content of the piece gradually narrows as notes are shared among the ensemble, until there's just a single note remaining. As a result every performance of Paragraph 7 is different in terms of its notes, its length and the overall sound. The outcome is random. As Cardew once pointed out, one of his standards was 'not to make a sound that's like something, but to make a sound that is just that …

204 Harris, Tony. *The Legacy of Cornelius Cardew*. (Farnham, Surrey: Ashgate, 2013): 66-67.

I want the feeling that everything you do is for the first time'.[205]

Iannis Xenakis and stochastic music

Although the above chance methods rely on random selection of one sort or another, none of them are as strictly mathematical as what became known as 'stochastic music'. Stochastic music was pioneered by the Greek composer Iannis Xenakis (1925 – 2001) who coined the term. In mathematics a stochastic process is one in which the steps are governed by rules of probability: a sequence of random objects often ordered in time and space. Xenakis used stochastic processes in his composition to determine musical parameters such as pitches, tempi, durations and timbres. Such numerical calculations were usually undertaken on computers. He wrote several articles and essays on stochastic processes, game-theory and computer programming in music. 1963 saw the publication of his seminal work on mathematics and music: *Formalized Music: Thought and Mathematics in Composition* (1971) in which he explains the thinking behind his techniques for composing music with stochastic mathematical functions.[206]

No discussion of the work of Xenakis, composer, architect and engineer, would be complete without mention of his extraordinary life. Of Greek parentage, he studied engineering in Athens and later, in Paris, composition with Olivier Messiaen. Messiaen advised him to take advantage of his architectural background 'You have the good fortune of being Greek, of being an architect and having studied special mathematics. Take advantage of these things. Do them in your music'. [207]Towards the end of World War II he fought for the Communist Greek resistance and in 1944 was severely injured in street fighting with British tanks. He was hit in the face by shrapnel receiving severe facial injuries and losing the sight in his left eye. When the new non-Communist government came to power in Greece, he was exiled to Paris under a death sentence which was not lifted until the end of the Greek junta in 1974. Following his arrival in Paris in 1947, he

205 Cited in Nyman, *Experimental Music*, 127.

206 Iannis Xenakis. *Formalized Music: Thought and Mathematics in Composition*. (New York: Pendragon Press, 1992).

207 Nouritza Matossian. *Xenakis*. (London: Kahn & Averill, 1986): 48.

worked for 12 years with the architect Le Corbusier. In 1958 Le Corbusier was commissioned to design the Philips Pavilion for the Brussels World Fair. He gave complete creative control of the project to Xenakis. As part of the World Fair, the Philips Pavilion staged a multimedia event in the Xenakis's steel and concrete structure which was based on parabolic curves. As visitors entered and exited the building they heard the music of Xenakis.[208] Once inside the auditorium *Poème* électronique by Edgar Varèse was projected from 350 speakers via 20 amplifier alongside ever-changing visuals creating a ground-breaking random simultaneity.

Xenakis's theories about stochastic synthesis were published in *Formalized Music, Thought and Mathematics in Composition*.[209] Here he advocated the use of stochastic processes to efficiently produce sonorities with 'numerous and complicated' transients arguing that the 'transient part of the sound is far more important than the permanent part in timbre recognition and in music in general'.[210] Stochastic processes are random and non-deterministic, that is, the next state of the environment is not fully determined by the previous one. Probability theory is used to determine what should happen next. In *Formalized Music*, Xenakis makes references to macroscopic (large scale) structures and the microscopic finer details of stochastic music. His music is often extremely elaborate in its detail, but that detail is usually at the service of the whole. This is well-illustrated in his orchestral piece *Pithoprakta* (1955-56) where Xenakis focuses on the overall block of sound, using probability theories to build different clouds of sound.[211] The finer detail, the individual notes and timbres are 'unpredictable' but are used to make up the combined mass of sound.[212]

Pithoprakta draws on probability theories including Gaussian distribution.[213] It is scored for string orchestra (with 46 individual parts), two trombones, xylophone, and wood block. The title translates as 'actions by means of probability'. The piece is based on the kinetic theory of gases

208 Xenakis' sound work *Interlude Sonore* which was subsequently titled *Concret PH*.

209 'New proposals in microsound structure' in Xenakis, *Formalized Music*, 242-254.

210 Xenakis, *Formalized Music*, 243-244.

211 Xenakis, *Formalized Music*, 12.

212 Gareth Roberts. *From Music to Mathematics {Exploring the connections}*. (Baltimore: John Hopkins University Press, 2016): 280.

213 Gaussian distribution (often known as normal distribution) is a very common continuous probability distribution and is the basis of a large proportion of statistical analysis.

where the temperature of a gas derives from the independent movement of its molecules. An analogy could be drawn between the movement of the molecules through space and that of the string instruments through their pitch ranges. At a macroscopic level, the work is made up of four sections which differ from each other in their use of textural and timbral characteristics, such as glissandi and pizzicato. As composer, Xenakis designed these large-scale features, but the individual components of sound such as which pitches and dynamics to use are generated by probability theories. Each sound-particle of the score is precisely defined through stochastic distribution functions at the same time as contributing to the overall sound impression. Xenakis created a graph representing a set of speeds of temperature and time; the pitches were drawn as the ordinates on the vertical scale and each of the instruments was represented by a jagged line representing a speed taken from the table of probabilities according to Gaussian distribution. The graph was then transcribed into traditional notation and the score was fixed. Continuous sounds are heard in the glissandi in the strings and trombones, for example, and discontinuous sounds include pizzicato plucking in the strings, tapping the strings with the opposite side of the bow, and the use of the wood block.[214]

In 1962, Xenakis composed his first string quartet *ST/4-1,080262*. The coded title translates as ST (stochastic composition), 4-1 (four instruments, first of its kind) created on *080262 (*8 February 1962). The work's material was calculated by an IBM 7090 Computer. The computer was programmed with compositional parameters that determine the point in time that a sound sequence occurs, next its timbre (arco, pizzicato, glissando and so on), choice of instrument, pitch, direction of glissando, duration of notes, and dynamics. The string quartet also made use of pioneering extended techniques such as playing with the wood of the bow, bowing the body of the instrument and other new tapping and plucking techniques. In 1963-4 the composition of *Eonta* for piano, two trumpets and three trombones was also based on the theory of probabilities where some of the instrumental parts were calculated on the IBM 7090 computer at the Place Vendome in Paris.

Xenakis used the idea of random walks for the creation of *Mikka* (1972), a short work for solo violin. A random walk is a process in which a sequence

214 Gareth Roberts lecture. 'Composing with Music. Iannis Xenakis and his Stochastic Music'. Gareth E. Roberts. Math/Music: Aesthetic Links, Montserrat Seminar, Spring 2012.

of discrete steps of fixed length is described in terms of the movement of a particle so in a one-dimensional random walk the state of the process is described as a position on a straight line. After starting at, say, 0 the first step takes the particle to either +1 or -1 with equal probability, after two steps it is +2, 0 or -2 and so on.[215] In *Mikka*, computer printouts of synthesized sounds are directly transformed into a continuous sequence of glissando curves for violin. The pitch is in continuous movement, with the left hand continuously sliding on the strings brushing past the microtones marked in the score. Plotted graphs of stochastic synthesis are also used in *N'Shima* (1975) and *Mikka S* (1975) where Xenakis read the horizontal axis of the graphs as time and mapped the vertical axis onto a grid of quarter-tone pitch values.

The music of *N'Shima* is based on the principle of Brownian motion, a continuous-time version of the random walk, displayed by minute particles of solid matter when suspended in a fluid or a gas resulting from their collision with the fast-moving molecules in the fluid. This translates in the music as short glissando figures sliding form one microtone to another. *N'Shima* is a Hebrew word meaning 'breath' or 'exhalation'. Scored for to mezzo sopranos and five instruments, the text of *N'Shima* text is taken from 'The Emperor's daughter and the King's son' a parable about two rival families whose children fall in love. Xenakis does not use whole words however; the vowels are used for tone colour and the consonants for articulation.

In the early 1980s, Xenakis's research, together with that of other colleagues specialising in electronics, software and signal processing, found a home at the *Centre d'Etudes de Mathématique et Automatique Musicales* in Paris along with the newly developed Unité Polyagogique Informatique du CEMAMu (UPIC) system completed in 1977. The UPIC system enabled composers to create sounds directly by 'moving an electromagnetic "pencil" across a sloping sensitised "drawing board"', and to hear the sounds they had created almost immediately. In the words of Richard Steinitz, the founder of the Huddersfield Contemporary Music Festival where the works of Xenakis were frequently performed, 'It thus united the visual and aural domains of Xenakis's career both as architect and composer'.[216]

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215 A two-dimensional random walk is represented by a set of steps along a grid or lattice. Brownian motion is a continuous-time version of the random walk.

216 Richard Steinitz. *Explosions in November: The First 33 years of Huddersfield Contemporary Music Festival*. (Huddersfield: University of Huddersfield Press, 2011): 91.

Each of the composers above combined mathematical devices with musical considerations in their chance compositions. The eighteenth-century dice composers manipulated and controlled the harmonic and melodic possibilities to ensure that whatever pieces arose 'by chance' would be musically convincing. In contrast, the aleatoric compositions of the twentieth century deliberately avoided musical control. Ives and Cowell experimented with episodes of chance and one of Cage's aims was to be 'free of individual taste and memory'. By the late sixties Cage's use of indeterminacy influenced many other composers amongst these Terry Riley whose piece *In C* could be regarded as a precursor of the Minimalism movement. However, the strictest mathematical approach was taken by Xenakis who coined the term stochastic music in which the compositional steps are governed by rules of probability.

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