Foreword

THE COMPOSER IGOR Stravinsky once referred to the way in which 'musical form is close to mathematics' in terms of 'mathematical thinking and relationships'. *Maths and Music* is divided into sections, each one exploring a different mathematical aspect of music: rhythm; symmetry; scales; form and structure; and finally some musical curiosities. Musical examples are taken from across the world, from ancient and medieval music through to popular music of the twenty-first century.

Chapter 1 explores the mathematical qualities of rhythm with examples of their application in Western classical music, popular music and jazz. Several twentieth-century composers took an innovative approach to rhythm and their work is examined in Chapter 2, looking in particular at pieces by Olivier Messiaen, John Cage, Steve Reich and Conlon Nancarrow. This is followed by a discussion of some of the rhythmic devices found in popular music and jazz including the use of unusual time signatures, disco beats and the rhythmic construction of electronic dance music.

The rhythms of Western music are relatively simple in comparison with the examples given of music from around the world in Chapter 3: the irregular divisions of aksak rhythms; the *tala* of North Indian classical music; the interlocking figurations of gamelan; the polyrhythmic patterns of West African drumming; and the beat cycles of Spanish flamenco. Euclidean rhythms are also introduced, a family of cyclical rhythms whose structures are derived from the Euclidean algorithm and whose use can be found in traditional music across the world.

Much Western music has its basis in repetition and contrast, so it is not surprising, that the different types of symmetrical transformation (reflection, rotation, translation, and scale) are frequently found in music.

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Chapters 4 and 5 look at the different ways in which musicians have used elements of symmetry and frieze patterns, ranging from the Medieval composer Guillaime de Machaut in 'Ma fin est mon commencement et mon commencement ma fin', to Nirvana in 'Smells like teen spirit'.

For centuries there has been much debate about different scales and tunings with many eminent mathematicians, scientists and musicians across Europe devising and analysing them. Three main systems evolved; the Pythagorean scale, just intonation and equal temperament and the mathematical and scientific principles involved are explored in Chapter 6; ratios, square roots and irrational numbers; the harmonic series and the circle of 5ths. Chapter 7 moves onto modes of limited transposition, such as the whole tone scale and the octatonic scale, along with the ubiquitous five-note pentatonic scale that is found in different forms across the world in music as diverse as the Javanese gamelan and the songs of The Smiths. The closing section of this chapter explores the use of the microtone, an interval smaller than a semitone that is used in scales which divide the octave into more than 12 notes.

Serialism with its note rows of 12 notes draws heavily on mathematical principles; it is algorithmic in its use of a strict set of procedures designed to be applied systematically; it uses symmetrical transformations in the construction of different rows and it uses combinatorics in its formulation of harmonies. These factors are explored in Chapter 8, primarily through the works of Schoenberg, Berg and Webern.

The focus then turns to form and structure, opening in Chapter 9 with two musical forms, canons and fugues, where rules and geometric transformations are central to their composition. In Stravinsky's words 'human activity must impose limits upon itself. The more art is controlled, limited, worked over, the more it is free'. Music is organized sound; an essential aspect of any musical composition is that the structure follows certain rules, but it should not be forgotten that sounds trigger an emotional response in the listener, understanding the mathematical structure does not tell us anything about the effect on the audience. Given that the set of procedures which need to be followed in a fugue could be described as an algorithm, perhaps a computer programme could be designed to fulfil this function; a discussion follows of the attempts that have been made to achieve this over recent decades.

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Golden Section and the associated Fibonacci series give rise to satisfying natural proportions. Their use in music is explored in Chapter 10 with examples as diverse as that of Stockhausen's *Klavierstuck IX* and Lady Gaga's song 'Perfect illusion'. It looks in particular at the music of Mozart, Satie, Bartók, Debussy, and composers associated with the Darmstadt School, questioning whether such ratios are used consciously by composers or simply stem from an intuitive sense of proportion.

Chapter 11 explores the different ways in which composers have used randomised elements in their music from the use of rolls of dice by eighteenth-century composers to the computerised chance operation of Iannis Xenakis who coined the term 'stochastic music'. The leading composer of aleatoric music was undoubtedly John Cage who used chance procedures in almost all the music he created after 1951.

Can fractals be found in music? Chapter 12 outlines various attempts that have been made to identify these patterns in music including the use of nested sequences and self-similarity. These include the prolation canons dating from the fifteenth century and two twentieth-century composers, Tom Johnson and György Ligeti. Two questions are posed throughout the book: did the composer use ideas from maths intentionally, and can you hear the mathematical nature? Some composers consciously based their music on mathematical principles: Iannis Xenakis used random numbers and stochastic processes; John Cage used what he called square root forms; and György Ligeti used the self-similarity and nested sequences of fractals in his compositions. These were all conscious links with mathematics made by twentieth-century composers, but that does not mean to say that the music of, say, J S Bach, Mozart and Beethoven, precludes any mathematical connections. Often the links with maths were used unconsciously; perhaps the closest link to fractals can be found in the prolation canons of Johannes Ockeghem which were composed hundreds of years ago at a time when the concept of fractals did not exist.

Finally, there is a section on various curiosities, mathematical techniques which do not fit neatly into the previous sections. Chapter 13 looks at examples of musical cryptography across the centuries, from the use of the solmization system by the Renaissance composer Josquin des Prez, through the monograms of various composers to the secret messages conveyed by Pink Floyd in their album *The Wall*. A number of composers have used

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magic squares and Latin squares to generate material for their compositions notably the two British twentieth-century composers Peter Maxwell Davies and John Tavener whose work is examined in Chapter 14. The book closes with the extraordinary case of change ringing, the practice of ringing church bells in a methodical order. Chapter 15 shows how this system pre-empted the mathematical discipline of group theory, a theory which was not properly established until nearly a century later.

In all, the book covers over 200 pieces of music ranging from classical symphonies to electronic dance music with numerous musical excerpts given. There are clear explanations throughout along with glossaries of musical and mathematical terms.

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