14 Magic squares and Latin squares

A NUMBER OF composers have used magic squares to generate material for their compositions. The first section of this chapter explores the way that magic squares have been used by three composers; the English composer Peter Maxwell Davies, and the French composers Pierre Boulez and Phillipe Manoury. The chapter then goes on to explore the use of Latin squares in music, in particular the ancient palindromic 5 x 5 Latin square known as the Sator Square which Anton Webern is thought to have used in his *Concerto Op. 24*. Inspired by the work of Webern, the English composer John Tavener also employed the Sator square in his music. The ways that he built it into his compositional technique are explored in two of his compositions; the song cycle *To a Child Dancing in the Wind* and his piece for double choir and string trio - *Ikon of Light*. The Italian composer Bruno Maderna used both Latin squares and magic squares as part of the creative process, notably in his *Serenata No. 2* which is looked at in some detail in the final section.

Magic squares

A magic square is a square matrix where a different positive integer is written in each cell, each occurring only once. The 'magical' quality is that each horizontal, vertical and diagonal row adds up to the same number. Here is a 3 x 3 magic square using the consecutive numbers 1 to 9 (Table 1). Each row and column adds up to 15 and each of the two diagonals also adds up to 15.

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Table 1 – Magic square of order 3

8	1	6
3	5	7
4	9	2

The constant number of the sum, in this case 15, is sometime referred to as the 'magical number' or the 'magic constant'.

The origins of magic squares are rooted in religious and esoteric contexts. One of the most famous magic squares is the Lo Shu 3 x 3 square (Table 1) which was known in China as early as 650 BCE. Legend has it that it was first seen on the back of a turtle emerging from the Lo river at a time of floods and the number pattern was seen as an omen. Every other 3 x 3 magic square can be derived from the Lo Shu by reflecting numbers of the square in the middle row or middle column and/or rotating numbers around the middle.

Magic squares have been a source of fascination for centuries and played a significant role in the development of numerology. Numerology attaches meaning and influence to numbers and number patterns. It is a belief in their divine, mystical power. Seven magic squares were dedicated to a planet or deity. The 3 x 3 square (Table 1) is known as the Square of Saturn. The 4 x 4 square is known as the Square of Jupiter where each row adds up to 34 (Table 2).

Table 2 – The Square of Jupiter

4	14	15	1
9	7	6	12
5	11	10	8
16	2	3	13

The others are:

Square of Mars 5 x 5
Square of Sol or the Sun 6 x 6
Square of Venus 7 x 7
Square of Mercury 8 x 8
Square of Luna or the Moon 9 x 9

Peter Maxwell Davies and magic squares

The British composer Peter Maxwell Davies (1934 - 2016) cultivated various styles of composition ranging from the monodrama *Eight Songs of a Mad King* (1969) which shocked audiences at its premiere, to a series of symphonies and occasional lighter pieces many of them inspired by Orkney (his place of residence) such as the ever-popular 'Farewell to Stromness'. His contribution to music education, particularly his part in the introduction of composing in schools, was significant. In 2004 he was made Master of the Queen's Music.

Davies used magic squares to generate material in several of his pieces. He was attracted partly by the internal rhythms of the squares and the way that they generated symmetric patterns, transpositions and inverted figures through both note values and pitch. His first piece based entirely on a magic square is *Ave Maris Stella* (1974). The composition is for six players and is based on a plainsong of the same name. Towards the end of the work, each aspect of the musical material is based on the 9 x 9 magic square - the Square of the Moon. The piece itself is in nine sections. He described the plainsong as being projected through the magic Square of the Moon and felt instinctively that the piece assumed some of the healing qualities associated with the square.

In his chamber piece *A mirror of whitening light* (1976-77) he uses the 8 x 8 Square of Mercury (Table 3).[268] The title makes reference to the alchemic process known as whitening when a base metal is transformed into gold. The Square of Mercury is associated with quicksilver and represents the perfect balance between the soul and the body. In Davies' words, the title

268 In various lectures and writings Maxwell Davies mistakenly referred to his use of the Square of the Sun.

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> refers to the mirror of whitening light outside my own window...there the bay is in fact like a crucible of ever changing miraculous light. I took as a starting-point the plainsong *Veni Sancte Spiritus* which, if one knows it, one can hear quite plainly recurring throughout the work. I also took as a principle of design the magic square … which I used as a kind of rhythmic and tonal grid-plan throughout the work…. the numbers 1 through 64 are arranged, on a square…they make very interesting patterns … the fact that these are numbers is of no consequence whatever. One is dealing in fact with rhythmic lengths and with pitches. For the listener, this is not of prime importance but I must mention it in passing to reassure the listener that the structure, underneath even the wildest passages, is very rigidly controlled.[269]

Table 3 – The Square of Mercury

8	58	59	5	4	62	63	1
49	15	14	52	53	11	10	56
41	23	22	44	45	19	18	48
32	34	35	29	28	38	39	25
40	26	27	37	36	30	31	33
17	47	46	20	21	43	42	24
9	55	54	12	13	51	50	16
64	2	3	61	60	6	7	57

The number 8 governs the whole structure of *A mirror of whitening light*. Here are some of the techniques Davies uses.[270]

The plainsong *Veni sancta spiritus* was used to generate much of the pitch material.

269 MaxOpus (http://www.maxopus.com/works/mirror.htm)

270 Jonathan Cross. 'Composing with numbers: sets, rows and magic squares' in *Music and Mathematics. From Pythagoras to Fractals*. (Oxford: Oxford University Press, 2003).

From the plainsong he derived this eight-note phrase G E F D F# A G# C.

Next he took the eight-note phrase and placed it down the furthermost left hand column (Column 1) and across the top row (Row 1).

Table 4

G	E	F	D	F#	A	G#	C
E							
F							
D							
F#							
A							
G#							
C							

Using the notes in Column 1 as the starting points, he then transposed the phrase seven times and inserted the transpositions into the 8 x 8 grid (Table 5).

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Table 5

G^{1}	E^{2}	F^{3}	D^{4}	F#^{5}	A^{6}	G#^{7}	C^{8}
E^{9}	C#^{10}	D^{11}	B^{12}	D#^{13}	F#^{14}	F^{15}	A^{16}
F^{17}	D^{18}	Eb^{19}	C^{20}	E^{21}	G^{22}	G^{23}	Bb^{24}
D^{25}	B^{26}	C^{27}	A^{28}	C#^{29}	E^{30}	Eb^{31}	G^{32}
F#^{33}	D#^{34}	E^{35}	C#^{36}	F^{37}	G#^{38}	G^{39}	B^{40}
A^{41}	F#^{42}	G^{43}	E^{44}	Ab^{45}	B^{46}	Bb^{47}	D^{48}
G#^{49}	F^{50}	F#^{51}	D#^{52}	G^{53}	A#^{54}	A^{55}	C#^{56}
C^{57}	A^{58}	A#^{59}	G^{60}	B^{61}	D^{62}	C#^{63}	F^{64}

The next step was to map the notes onto the Square of Mercury matrix in the order that the numbers appear (Table 6).

Table 6

C^{8}	A^{58}	A#^{59}	F#^{5}	D^{4}	D^{62}	C#^{63}	G^{1}
G#^{49}	F^{15}	F#^{14}	D#^{52}	G^{53}	D^{11}	C#^{10}	C#^{56}
A^{41}	G^{23}	G^{22}	E^{44}	Ab^{45}(D)	Eb^{19}	D^{18}	D^{48}(Ab)
G^{32}	D#^{34}	E^{35}	C#^{29}	A^{28} (D)	G#^{38}	G^{39}	D^{25}(A)
B^{40}	B^{26}	C^{27}	F^{37}	C#^{36}	E^{30}	Eb^{31}	F#^{33}
F^{17}	Bb^{47}	B^{46}	C^{20}	E^{21}	G^{43}	F#^{42}	Bb^{24}
E^{9}	A^{55}	A#^{54}	B^{12}	D#^{13}	F#^{51}	F^{50}	A^{16}
F^{64}	E^{2}	F^{3}	B^{61}	G^{60}	A^{6}	G#^{7}	C^{57}

Notice that four of the numbers were altered by Davies at his discretion (and with no explanation); 45 and 48 have been interchanged, as have 25 and

28. These have been shown in brackets.[271] Having created the new matrix Maxwell Davies then took different routes through it to generate melodic lines, the 'very interesting patterns' he refers to: left to right across; right to left across; top to bottom; bottom to top; diagonals and spirals.[272] In this way the notes produced would be, for example,

left to right across C A A# F# D D C# G

top to bottom C G# A G B F E F

The first two instruments to play in *A mirror of whitening light* are the trumpet and the flute. Their pitches are taken from the top two lines of Table 6 (the first 13 notes). They use the following pitches

Trumpet C A A# F# D

At times throughout the piece Davies uses enharmonic equivalents so the trumpet part is notated as

271 Gareth Roberts. *From Music to Mathematics {Exploring the connections}*. (Baltimore: John Hopkins University Press, 2016): 263.

272 The manipulation of these rows and columns bears some similarity to the way that the note row is treated in Serialism (see Chapter 8).

C A Bb Gb D[273]

Flute D C# G G# F F# D# G
This is notated as D C# G Ab F Gb Eb G

In his analysis of *A mirror of whitening light*, the musicologist Peter Owens gives further examples of the ways in which Davies traced pathways through the magic square in order to generate pitches. The word '*Hauptstimmen*' is used to denote the principal parts.[274]

For each of the work's main subsections, beginning at rehearsal letters F, J, Q, W, Z, El, H1 and two bars after J1, *Hauptstimmen* are created by tracing pathways through the matrix so that all cells are realised without repetition or omission. Bassoon and cor anglais, for example, share a presentation of the rows of the matrix - all read left to right - between F and J; violin 2 presents its columns - alternating ascending and descending readings - between J1 and Q1; other realisations trace zig-zags through the diagonals, or 'L' shapes, all, significantly, beginning and ending in different corner cells of the matrix, two of which contain the pitch class C.[275]

The Square of Mercury is also utilised at points to generate rhythms. The numbers in the Square of Mercury go from 1 to 64 (see Table 3). The range of note values is clearly impractical i.e. if number 1 was allocated a semiquaver

273 Notes that sound the same but are written (or 'spelt') differently are said to be enharmonic e.g. E# and F where E# is the enharmonic equivalent of F.

274 *Hauptstimme* is the name given by the serialist composers Schoenberg and Berg to a principal part in a complex texture, usually in serial or other rigorously non-tonal music.

275 Peter Owens .'Revelation and Fallacy: Observations on Compositional Technique in the Music of Peter Maxwell Davies'. *Music Analysis* , July - October, 1994, Vol. 13, No. 2/3: 161-202.

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(sixteenth note), 2 a quaver (eighth note), 3 a dotted quaver (dotted eighth note) and so on, then the number 64 would be the equivalent of two breves. In order to overcome this potential problem, Davies used modular arithmetic to convert the numbers to those between 1 and 8. Modular arithmetic gives the remainder when dividing the magic square numbers by the eight. So, for example, 58/8 is 7 x 8 remainder 2 and 43/8 is 5 x 8 remainder 3. In mathematical notation this is shown as

$58 \equiv 2 \pmod{8}$ and $43 \equiv 3 \pmod{8}$

The value of a quaver is then allocated to each integer so that

1= a quaver (eighth note)
2 = a crotchet (quarter note)
3 = a dotted quaver (dotted eighth note)
4 = a minim (half note)
5 = a minim tied to a quaver (half note tied to an eighth note)
6 = a dotted minim (dotted half note)
7 = a dotted minim tied to a quaver (dotted half note tied to an eighth note).

There is a slight flaw in this system in that 8, 16, 24, 32, … 64 are all divisible by 8 so there is no remainder. So 8 takes the value of 8 quavers and becomes a semibreve. The resultant Square of Mercury can be seen in Table 7 which takes into account the alterations Davies made in his original transcription of the Square of Mercury (Table 6). It also assumes that a rest following a note counts towards it, a strategy that gave Davies more creative freedom.[276]

276 Roberts. *From Music to Mathematics*, 265.

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Table 7

8	2	3	5	4	6	7	1
1	7	6	4	5	3	2	8(5)
1	7	6	4	5(8)	3	2	8
8	2	3	5	4(1)	6	7	1(4)
8	2	3	5	4	6	7	1
1	7	6	4	5	3	2	8
1	7	6	4	5	3	2	8
8	2	3	5	4	6	7	1

Cross (2003) questions whether any of this can be heard when listening to *A mirror of whitening light* noting Davies' comment that the 'sequences of pitches and rhythmic lengths … [are] easily memorable once the "key" to the square has been found'. Although Cross observes that Davies might argue that the 'logic' given to the transformations may be 'subconsciously perceived' he goes on to state his own doubts as to this possibility.[277]

Pierre Boulez and magic squares – *Structures 1a*

In the early 1950s, several composers including Pierre Boulez (1925-2016) extended the compositional procedures of serialism to the other aspects of the music beyond pitch, a method which has become known as total serialism or integral serialism (see pages 133-4). Boulez based his piece *Structures 1a* (1951) entirely on a note row taken from Messiaen's piano piece *Mode de valeurs et d'intensités* (Mode of Durations and Intensities) (1950). From this he created two number matrices which represented all 48 versions of the row. These can be represented as a pair of magic squares (Table 8 and Table 9). In these squares, the number 1 always refers to Eb (pitch 1 of the original series) and to a demi-semiquaver (thirty-second note) - the first degree of the chromatic scale of durations. The number 2 always refers to D (pitch 2 of the

277 Cross, 'Composing with numbers', 140.

original series) and to a semiquaver (sixteenth note) - the second degree of the durational scale, and so on.

Table 8

1	2	3	4	5	6	7	8	9	10	11	12
2	8	4	5	6	11	1	9	12	3	7	10
3	4	1	2	8	9	10	5	6	7	12	11
4	5	2	8	9	12	3	6	11	1	10	7
5	6	8	9	12	10	4	11	7	2	3	1
6	11	9	12	10	3	5	7	1	8	4	2
7	1	10	10	3	5	11	2	8	12	6	9
8	9	5	4	5	7	2	12	10	4	1	3
9	12	6	11	7	1	8	10	3	5	2	4
10	3	7	1	2	8	12	5	5	11	9	6
11	7	12	10	3	4	6	1	2	9	5	8
12	10	11	7	1	2	9	3	4	6	8	5

Table 9

1	7	3	10	12	9	2	11	6	4	8	5
7	11	10	12	9	8	1	6	5	3	2	44
3	10	1	7	11	6	4	12	9	2	5	8
10	12	7	11	6	5	3	9	8	1	4	2
12	9	11	6	5	4	10	8	2	7	3	1
9	8	6	5	4	3	12	2	7	11	10	7
2	1	4	3	10	12	8	7	11	5	9	6
11	6	12	9	8	2	7	5	4	10	1	3
6	5	9	8	2	1	11	4	3	12	7	10
4	3	2	1	7	11	5	10	12	8	6	9
8	2	5	4	3	10	9	1	7	6	12	11
5	4	8	2	1	7	6	3	10	9	11	12

If the twelve columns in Table 8, reading from left to right across or down from top to bottom, represent the twelve possible transpositions of the original row, then the same columns read from right to left or bottom to top represent the twelve possible retrogrades. Similarly the columns of numbers in Table 9, (from left to right or top to bottom) represent the twelve inverted row forms, and (from right to left or bottom to top) the retrograde-inversions.

Phillipe Manoury - the Square of Jupiter in *Melencolia (d'apres Dürer)*

In medieval alchemy special qualities were assigned to each of the magic squares defining the relationship between the soul and the body. Each square was associated with a metal as well as its corresponding planet. The Square of Jupiter is associated with tin and represents the soul emerging from the body. The German artist Albrecht Dürer featured a 4 x 4 square (Table 10) with the magic number of 34 in his engraving *Melencolia I* of 1514. As can be seen, this is a variant on the Square of Jupiter.

Table 10

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

Notice how the date 1514 is embedded on the bottom row and the way in which the four squares at the centre and each of the four 2 x 2 square grids at each corner also add up to 34. The French composer Phillipe Manoury (b. 1952) used Dürer's engraving as part of the basis for his Third String Quartet *Melencolia (d'apres Dürer)* (2013). He used the tabulated data for generating collections of pitch classes – often tetrads (a set of four notes) – and sometimes used it for other purposes, such as creating rhythmic patterns.[278]

Latin squares

Another type of square grid (or matrix) is the Latin square: a square array of symbols arranged in rows and columns such that each row or column of the array contains each symbol precisely once. In mathematical terms this would be described as an $n \times n$ array of order n with entries from a set of n distinct symbols. So if $n = 4$ then a Latin square of order 4 using letters from the alphabet could be as seen below in Table 11. The word 'Latin' refers to the symbols which were taken from the Latin alphabet.

Table 11 – Latin square of order 4

A	B	C	D
B	A	D	C
C	D	A	B
D	C	B	A

Latin squares have proved to be useful in statistical experiments as well as to construct codes. They were discovered by the Swiss mathematician Leonhard Euler in 1779. Probably the most familiar use of a Latin square is in Sudoku (meaning 'single digit') where a skeleton, usually of order 9, is given containing a few numbers. The challenge is to fill in the blanks with the remaining numbers so that that each row or column of the array contains each number only once.

278 Besada, J.L., Guichaoua, C. and Andreatta, M. (2022), From Dürer's Magic Square to Klumpenhouwer Tesseracts: On Melencolia (2013) by Philippe Manoury. *Music Analysis*, 41: 145-182.

Anton Webern and Latin squares

In his book *The Path to New Music* (1933), Anton Webern points out that there is a linguistic analogy between serialism and the ancient palindromic 5 x 5 Latin square known as the Sator Square (see Table 14) although he mistakenly refers to this as a magic square. The words translate loosely as 'Arepo[279] the sower holds the wheels for his work'.[280] The square can be read horizontally, vertically and backwards so it is easy to see the similarity with the manipulation of note rows in serialism – inversion, retrograde and retrograde inversion (see Chapter 8).

Table 14 – The Sator Square

S	A	T	O	R
A	R	E	P	O
T	E	N	E	T
O	P	E	R	A
R	O	T	A	S

David Cohen argues that the trichords found in the final 15 bars of Anton Webern's *Concerto Op. 24* (1934) are based on a Latin square. Although Webern's sketches give little indication of how he arrived at these trichords, Cohen argues that there seems little doubt that he was attempting to 'find a musical counterpart to the intricate structural relationship of the word square' given that the sketches of the work show several attempts to arrange the words of the palindrome under the trichords.[281]

279 The meaning of the word Arepo is unclear, there are several interpretation including the possibility that it was simply a residual word needed to complete the square.

280 Geoffrey Haydon. *John Tavener. Glimpses of Paradise*. (London: Victor Gollancz, 1995): 164.

281 David Cohen. 'Anton Webern and the Magic Square'. *Perspectives of New Music*, Vol. 13, 1974: 213-215.

John Tavener and the Sator Square

The music of the English composer John Tavener (1944-2013) is imbued with religious symbolism; he was originally inspired by the Roman Catholic faith and later by Orthodox Christianity in works such as *The Protecting Veil*. Some years after Webern had encountered Latin squares, Tavener came across a copy of Hans Moldenhauer's chronicle of Webern's life and work wherein he found a diagram of the Sator Square.[282] The first piece in which Tavener used the Sator Square, or the 'Byzantine palindrome' as he referred to it, was in the song cycle *To a Child Dancing in the Wind* (1983). He took the square and worked out a system whereby he read it horizontally and assigned a number to each of the eight different letters.[283]

SATOR AREPO TENET OPERA ROTAS

The letters of SATOR were allocated the numbers 1 2 3 4 5, the new letters introduced in AREPO (E and P) become 6 and 7. The final letter is the N found in TENET, so the square becomes as shown below.

1	2	3	4	5
2	5	6	7	4
3	6	8	6	3
4	7	6	5	2
5	4	3	2	1

Tavener used a similar system the following year in his seven-movement work for double choir and string trio - *Ikon of Light*. The piece was largely inspired by the prayer *Invocation to the Holy Spirit* by St Symeon (949–1022). For the top line of his square Tavener chose a five-note phrase based on the Greek word *Elthe* (Come). The five notes are based on a descending pattern starting on the first note and moving down in step. To extend this range he drew up a second square where five notes ascend from the first note, the same

282 Hans Moldenhauer. *Anton Von Webern: A Chronicle of His Life and Work*. (London: Random House, 1979): 431.
283 Haydon, *John Tavener*, 164-165.

point. The first movement has five sections where the first section follows the pattern SATOR, ROTAS – SATOR, AREPO – SATOR, OPERA – and finally SATOR, TENET. This is not always obvious on listening because of Tavener's use of elaborations, extensions and microtonal decorations.[284] The second section for string trio is easier to follow on hearing; it follows the pattern SATOR, AREPO, TENET, OPERA, ROTAS with long silences between each phrase. Each of the other movements uses different manipulations of the Sator Square.[285]

Bruno Maderna and Latin squares – *Serenata No. 2*

The Italian composer and conductor Bruno Maderna (1920–73) used both magic squares and Latin squares (as well as other arrays) as part of the creative process in the construction of his music. Maderna was a central figure at the Darmstadt Summer School in the 1950s and his music was largely based on the principles of serialism (see Chapter 8). The arrays he designed were in response to what he perceived to be the shortcomings of serialism and the Second Viennese School (see page 125). In his words

> What I did not like about twelve-tone theory is the principle by which, once a series is given, it has to reappear in its entirety, continuously, vertically and horizontally, for the sake of consistency in the musical discourse. … Slowly but surely I proceeded to the study of serial [permutation, until I succeeded in developing more and more complex and rigorous systems of mutation.[286]

Serenata No. 2 (1954, revised 1957) is scored for eleven instruments. The first part of the work uses 11-note serial arrays rather than the usual 12 notes used in standard note rows (the note Bb is omitted). The second part of the *Serenata* is divided into four sections, each of them using a different set of nine pitch classes. The permutations of both the pitch classes and rhythmic material are determined by the two 9 x 9 squares below. Table 12 Is a Latin

284 Microtones are intervals that are smaller than a semitone.

285 For more details see Haydon, *John Tavener*, 172-173.

286 Cited in Christoph Neidhofer 'Bruno Maderna's Serial Arrays'. MTO a journal of the Society for Music Theory Vol. 13, No. 1, March 2007.

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square based on the first nine letters of the alphabet. Maderna used this to determine the pitch-class permutations.

Table 12 – Latin square used by Maderna to determine pitch classes in *Serenata No. 2*

D	C	A	B	G	H	F	E	I
H	F	C	D	1	A	E	B	G
I	A	B	G	E	D	H	C	F
E	I	G	H	C	F	B	A	D
B	H	F	E	A	G	D	I	C
F	E	D	C	B	I	G	H	A
A	G	E	I	D	B	C	F	H
C	D	H	A	F	E	I	G	B
G	B	I	F	H	C	A	D	E

Table 13 is a magic square containing the values 1 to 81 once each. The values in each row, column, and diagonal add up to the same sum of 369. This magic square is used to generate the rhythmic structure of each of the four sections. The values of the durations were calculated using (semi-quavers) sixteenth notes; the numbers in the magic square were multiplied by 1/16 to calculate the overall duration so, for example, the number 8 corresponds to a minim (half note), the number 16 corresponds to a semibreve (whole note) and so on.[287]

Table 13 – Magic square used by Maderna to generate the rhythmic structure in *Serenata No. 2*

37	78	29	70	21	62	13	54	5
6	38	79	30	71	22	63	14	46
47	7	39	80	31	72	23	55	15
16	48	8	40	81	32	64	24	56
57	17	49	9	41	73	33	65	25

287 For further details of how Maderna applied these techniques to *Serenata No. 2* and other works see Neidhofer 'Bruno Maderna's Serial Arrays'.

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26	58	18	50	1	42	74	34	66
67	27	59	10	51	2	43	75	35
36	68	19	60	11	52	3	44	76
77	28	69	20	61	12	53	4	45

The following example shows how Maderna has assigned the nine pitch classes I to the letters A to I in bars 183 – 200.

Next these pitch classes were entered into the Latin square of Table 13 (rotated 90 degrees anti-clockwise) so that each row, column, and diagonal of the array contained each of the nine pitch classes exactly once (Table 14).

Table 14

I	G	F	D	C	A	H	B	E
F	B	C	A	I	H	F	G	D
F	E	H	B	D	G	C	I	A
H	A	D	F	G	I	B	E	C
G	I	E	C	A	B	D	F	H
B	D	G	H	E	C	I	A	F
A	C	B	G	F	D	E	H	I
C	F	A	I	H	E	G	D	B
D	H	I	E	B	F	A	C	G

I^{57}	G^{16}	F^{47}	D^{26}	C^{17}	A^{48}	H^{7}	B^{67}	E^{58}

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The numbers indicate the path Maderna has chosen through Table 13.

Taking the first row across, this means that the pitches are

E	D	C#	Bb	Ab	F	Eb	G	B

The durations are calculated by multiplying the numbers in Table 13 by 1/16.

So the music in bars 183 – 188 of *Serenata No. 2* is as follows.

This may appear to some as a rather clinical way of composing music, objective rather than subjective. Maderna argued that the use of objective procedures in his music, such as the manipulation of Latin squares and magic squares, had no expressive quality but once realized they led to the imaginative qualities of his compositions. As he wrote,

I have my own grammatical system. It arises from the principle of serialism and is sufficiently flexible. Above all it is sufficiently abstract to allow me complete freedom in realizing my musical imagination - which is not at all abstract – in a thousand ways.[288]

288 1965. 'La révolution dans la continuité.' *Preuves* 15/177: 28–29.

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